

**A dissertation submitted to Huazhong University of Science and  
Technology for the Degree of Doctor of Philosophy in Science**

**Transport Properties of Fractal Tree-like  
Branching Network**

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## **Abstract**

The tree-like branching network structures abound in both living and non-living systems, such as tree, leaves, mammalian circulatory and respiratory systems, neural dendrites, river basins, deltas, lightning, oil/water reservoir, streets, rapid solidification etc. The efficient transport characteristics of natural branching systems can provide useful hints for optimal solutions of many engineering problems, which have fascinated researchers in physics, chemistry, biology, physiology, engineering and geology for ages and been applied in microelectronic cooling, flow-field designs in fuel cell, microfluidic manifolds in lab-on-a-chip systems etc. The tree-like branching network has been shown to be a conventional self-similar fractal network and the ideas of fractal geometry can be implemented to study the network. The objective of the current research is to investigate the transport properties of tree-like branching network with fractal characteristics and the effect of microstructures on the transport properties of network, which will shed light on mechanism of natural network and application in engineering.

This thesis is organized as six Chapters. In Chapter 1, the progress of tree-like branching network, fractal and transport theory, and general description of fractal tree-like branching network are introduced and reviewed. In Chapter 2, Murray's law, which is the fundamental in this field, and its generalized forms are addressed, and the single branching structure is optimized by minimizing resistance under fixed volume for different transport processes. Chapter 3 focuses on the thermal properties of fractal tree-like branching network. In this chapter, the effective heat conductivity is derived, the heat conduction of composite material embedded with H shaped branching network is studied, and the disc-shaped branching network with/without loops is simulated with CFD for heat convection. In Chapter 4, the laminar flow and seepage flow in two kinds of fractal tree-like branching network are investigated, the radial effective permeability

of heterogeneous porous media is calculated with the proposed dual-domain model of fractal tree-like branching network and homogeneous porous medium model. In Chapter 5, the scaling laws between various transport parameters and volume or surface area are derived. Finally, a summary of the present work is given in Chapter 6, and some potential research interests are commented for future study of tree-like branching network and its applications.

**Keywords:** Tree-like branching network, Fractal, Murray's Law, CFD, Heat and mass transfer, Scaling law, Porous media