CHAPTER 17

Dimensional Analysis

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17.1 INTRODUCTION

Dimensional analysis is a mathematical tool that is used to reduce complex physical problems to the simplest forms before quantitative analysis and experimental investigation are carried out. The reduction number of variables uses the Buckingham theorem[1] as its central tool. In addition, it allows identification of dimensionless parameters even if the form of equation is still unknown.

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Therefore, it reduces, by an order of magnitude, the number of experiments needed to generalize or empirically correlate a set of data.

Dimensional analysis, in its most primitive form, is used to check the validity of algebraic derivations or equations. Every term or quantity in a physically meaningful expression or equation has the same dimension, and they can be added and subtracted from one another. Both sides of any expression must have the same dimension.

In engineering, empirical results obtained from experiments are sometimes difficult to present in a readable form such as in the form of graphs. Here, dimensional analysis provides a way to choose the relevant data and to present them concisely. This, in turn, helps the development of theoretical modeling of the problem. Relationships between influencing factors can be determined, generalization of experimental data can be performed. It is useful for predicting performance of different systems.[2,3]

Its application in science and engineering is ubiquitous, including its use in food processing. It is an economical way to scale up processes as dimensional analysis reduces the degree of freedom of the physical problems to the minimum. Dimensional analysis is a useful tool in engineering experimentation and analysis, modeling, design, and scale up. This technique is extensively used in other fields, although the focus here is on engineering, scientific, and technological applications.

17.2 LIMITATIONS

One important limitation of dimensionless analysis is that it does not unravel the underlying physics or the nature of a physical phenomenon. Therefore, variables that affect or influence the phenomenon should be known prior to dimensional analysis. It should be noted that selection of variables is vital in ensuring a successful dimensional analysis. Therefore, it is important to know a priori the relevance of parameters to be included in such analysis. Sometimes, it may be necessary to carry out an iterative process by including or excluding certain parameters and correlating the resulting dimensionless groups using experimental data.

17.3 HOW TO OBTAIN DIMENSIONLESS NUMBERS

Dimensionless numbers can be derived by variables and parameters governing any process from a number of methods. Two most common methods in use are dimensionless analysis of the differential conservation equation and the Buckingham Π method. There are alternative avenues for the generation of dimensionless groups as well, but they are beyond the scopes of this concise chapter.

17.3.1 Buckingham's Π Theorem

Buckingham's first theorem states that the number of independent dimensionless numbers, m, that can be formed is equal to the total number of the physical quantities, n, minus the number of primary dimensions, r, that are used to express dimensional formulas of the physical quantities in question. In other words, every physical relationship between n physical quantities can be reduced to a relationship between m mutually independent dimensionless numbers.[1–3]

If a physical problem is expressed by n independent physical quantities, Q_i

$$f(Q_1, Q_2, Q_3, \ldots, Q_n) = 0,$$

then, according to the Buckingham's first theorem, this can also be expressed by $(n^N m$ independent dimensional numbers, π_i, as

$$f(\pi_1, \pi_2, \pi_3, \ldots, \pi_{n-m}) = 0.$$

Buckingham's second theorem states that each π group is a function of core variables (also known as *governing* or *repeating* variables) plus one of the remaining variables. Core variables are those variables that will most probably appear in all or most of the π groups. According to the second theorem, only m core variables are to be chosen. Core variables can be freely chosen by following the rules:

- A combination of the core variables must contain all basic dimensions.
- A combination of the core variables must not form a dimensionless group.
- The core variable does not necessarily appear in all π groups.
- The core variables that are chosen must be measurable in an experimental investigation that is of major interest to the designer.

It should be noted that if extra unimportant variables are introduced, then extra π groups will be formed. They will play small roles in influencing the physical behavior of the problem. However, if an important influential variable is missed, then a π group will be missing. Experimental analysis based on these results may miss significant behavioral changes. Therefore, it is very important to choose all influencing factors when a relevance list is made.

A relevance list consists of all influencing dimensional parameters. The parameters consist of a target quantity, geometric parameters, material parameters, and process-related parameters. In each case, only one target quantity is to be considered, and it is the only dependant parameter. The rest of the parameters in the list are independent of one another.[4,5]

Generally, the quantities and parameters are lished in a bracket { }. Semicolon separates the quantities and parameters into various categories which are mentioned below. Therefore, a reference list is normally written in the following form:

{target quantity; geometric parameters; material parameters; process-related parameters}

For an incompressible fluid of density ρ and viscosity μ which is flowing through a tube of length L at velocity v, the pressure drop over the tube length is ΔP. The relevance list is constructed based on the relevant physical quantities mentioned above.

In this example, the relevance list is

$$\{\Delta P; L; \rho, \mu; v, g\}.$$

Note that ΔP is the target quantity, L is the geometric parameter, P and μ are the material parameters, v and g are the process related parameters.

Fundamental dimensions that made up the quantities listed in the relevance list are length, time, and mass; therefore, r is 3, the number of physical quantities in question, n, is 6; consequently, the number of dimensionless groups, m, that can be formed is 3. Three variables are to be chosen as the core variables common to all three dimensionless groups. In this case, L, v, and ρ are selected. The three dimensionless groups (also known as π groups) are

$$\pi_1 = L^a v^b \rho^c g^1,$$
$$\pi_2 = L^d v^e \rho^f \Delta P^1,$$
$$\pi_3 = L^g v^h \rho^i \mu^1.$$

If expressed in terms of basic dimensions,

$$[\pi_1] = (L)^a (Lt^{-1})^b (ML^{-3})^c (Lt^{-2})^1,$$

$$[\pi_2] = (L)^d (Lt^{-1})^e (ML^{-3})^f (ML^{-1}t^{-2})^1,$$

$$[\pi_3] = (L)^g (Lt^{-1})^h (ML^{-3})^i (ML^{-1}t^{-1})^1.$$

To make the π groups dimensionless, the physical quantities must be raised to certain exponents, a, b, c, and so forth. To evaluate the exponents, let the dimension of the π groups be zero.

Π_1	$L^0t^0M^0 = L^{a+b-3c+1}t^{-b-2}M^c$
Π_2	$L^0t^0M^0 = L^{d+e-3f-1}t^{-e-2}M^{f+1}$
Π_3	$L^0t^0M^0 = L^{g+h-3i-1}t^{-h-1}M^{i+1}$

Next, these exponents are equated at both sides:

Π_1	For dimension L: $0=a+b-3c+1$ For dimension t: $0=-b-2$ For dimension M: $0=c$ Hence, $c=0$; $b=-2$; $a=1$
Π_2	For dimension L: $0=d+e-3f-1$ For dimension t: $0=-e-2$ For dimension M: $0=f+1$ Hence, $f=-1$; $e=-2$; $d=0$
Π_3	For dimension L: $0=g+h-3i-1$ For dimension t: $0=-h-1$ For dimension M: $0=i+1$ Hence, $i=-1$; $h=-1$; $g=-1$

Substituting these exponents values into the π groups equations yield the following dimensionless groups:

$$\pi_1 = L^1 v^{-2} g^1 \text{ gives } \pi_1 = \frac{gL}{v^2} \text{ that is the reciprocal of the Froude number;}$$

$$\pi_2 = v^{-2}\rho^{-1}\Delta P^1 \text{ gives } \pi_2 = \frac{\Delta P}{\rho v^2} \text{ that is the Euler number;}$$

$$\pi_3 = L^{-1}v^{-1}\rho^{-1}\mu^1 \text{ gives } \pi_3 = \frac{\mu}{Lv\rho} \text{ that is the reciprocal of the Reynolds number.}$$

Another way to obtain dimensionless numbers in the Buckingham method is to use dimensional matrix to determine the dimensionless numbers. The columns of the dimensional matrix are assigned to the individual physical quantities and the rows are assigned to the exponent values.

The dimensional matrix is subdivided into a quadratic core matrix and a residual matrix.[5,6]

Dimension	L	v	ρ	g	ΔP	μ
L	1	1	−3	1	−1	−1
T	0	−1	0	−2	−2	−1
M	0	0	1	0	1	1
L+T	1	0	−3	−1	−3	−2
−T	0	1	0	2	2	1
M	0	0	1	0	1	1
(L+T)+3M	1	0	0	−1	0	1
−T	0	1	0	2	2	1
M	0	0	1	0	1	1
	Core			Residual		

Quadratic core matrix (3 x 3)

Residual matrix (3 x 3)

Identity matrix (3 x 3)

To obtain the dimensionless numbers, each quantity in the residual matrix forms the numerator of a fraction, whereas its denominator consists of the fillers from the core matrix (that has been transformed into identity matrix by mathematical manipulation) with exponents indicated in the residual matrix. The π groups are

$$\pi_1 = \frac{g}{L^{-1}v^2\rho^0} \text{ gives } \pi_1 = \frac{gL}{v^2} \text{ that is the reciprocal of the Froude number;}$$

$$\pi_2 = \frac{\Delta P}{L^0\rho^1 v^2} \text{ gives } \pi_2 = \frac{\Delta P}{\rho v^2} \text{ that is the Euler number;}$$

$$\pi_3 = \frac{\mu}{L^1 v^1 \rho^1} \text{ gives } \pi_3 = \frac{\mu}{Lv\rho} \text{ that is the reciprocal of the Reynolds number.}$$

Both methods of the Buckingham theorem give a Froude number, Fr, an Euler number, Eu, and a Reynolds number, Re.

17.3.2 Dimensional Analysis of Governing Differential Equations

Differential equations are often derived from first principles to describe various transport phenomena. Essentially, they are equations of conservation of mass, momentum, species, and energy. Dimensional homogeneity requires that every term in a physically meaningful differential equation has the same units or dimensions. The ratio of one term in the equation to another term is necessarily dimensionless, and the ratio is known as dimensionless number (or group). Therefore, the interpretation of the dimensionless number is clear if one knows the physical meaning of each term in the equation.

For example, the x-component of the Navier–Stokes equation for laminar steady flow of an incompressible Newtonian fluid is

$$\vec{v}_x \frac{\partial \vec{v}_x}{\partial x} + \vec{v}_y \frac{\partial \vec{v}_y}{\partial y} + \vec{v}_z \frac{\partial \vec{v}_z}{\partial z} = g_x - \frac{\partial P}{\rho \partial x} + \frac{\mu}{\rho}\left(\frac{\partial^2 \vec{v}_x}{\partial x} + \frac{\partial^2 \vec{v}_y}{\partial y} + \frac{\partial^2 \vec{v}_z}{\partial z}\right) \tag{17.1}$$

Equation 17.1 can be expressed in terms of dimensional equality as Equation 17.2. Each term in Equation 17.2 has the dimension of [L/t^2], viz. acceleration.

$$\left[\frac{\vec{v}^2}{L}\right] = [g] - \left[\frac{P}{\rho L}\right] + \left[\frac{\mu \vec{v}}{\rho L^2}\right] \tag{17.2}$$

The physical meaning of each term in Equation 17.2 is commonly given as

$$[\text{inertia force}] = [\text{gravity force}] - [\text{pressure force}] + [\text{viscous force}].$$

The equal sign here is not correct, however. This equation simply represents a balance between various forces acting on a fluid particle in the flow system.

There are six ratios that can be formed from the four terms shown in Equation 17.2, namely

$$\frac{[\vec{v}^2/L]}{[g]}, \frac{[\vec{v}^2/L]}{[P/\rho L]}, \frac{[\vec{v}^2/L]}{[\mu v/\rho L^2]}, \frac{[g]}{[P/\rho L]}, \frac{[g]}{[\mu \vec{v}/\rho L2]}, \text{and}, \frac{[P/\rho L]}{[\mu \vec{v}/\rho L2]},$$

that gives the following dimensionless numbers:

$$\frac{\vec{v}^2}{gL}, \frac{\rho \vec{v}^2}{P}, \frac{L\vec{v}\rho}{\mu}, \frac{L\rho g}{P}, \frac{L^2\rho g}{\mu \vec{v}}, \frac{PL}{\mu \vec{v}}.$$

Dimensionless numbers or their reciprocals derived this way have clear physical meaning. Some of the well-known dimensionless groups obtained from the Navier–Stokes equation are given in Table 17.1. Note that products of dimensionless groups as well as ratios of such groups raised to any integral power are also dimensionless. However, they have no physical meaning in general and are rarely used. It is important to note that the numerical values of some dimensionless numbers can lead to misleading interpretation of the physics involved, so care must be exercised in attributing significance to these numbers.

17.3.3 Dimensional Analysis on Transport Equations

The same method shown in the above example is performed on mass, heat, and momentum transport equations, respectively. Note that to obtain dimensionless heat- and mass-transfer coefficients, one needs to cast the convective boundary conditions in a special form so that the boundary

Table 17.1 Dimensionless Numbers Obtained from Dimensional Analysis of Navier–Stokes Equations

Dimensionless Number in Equation 17.4	Interpretation	Formula	Symbol	Dimensionless Group
$\vec{v}^2/gL$	Ratio of inertia force to gravity force	$\vec{v}^2/gL$	Fr	Froude number
$\rho\vec{v}^2/P$	Ratio of inertia force to pressure force	Reciprocal = $P/\rho\vec{v}^2$	Eu	Euler number
$L\vec{v}\rho/\mu$	Ratio of inertia force to viscous force	$L\vec{v}\rho/\mu$	Re	Reynolds number
$L\rho g/P$	Ratio of gravity force to pressure force	$P/L\rho g$	Eu·Fr	Product of Euler number and Froude number
$L^2\rho g/\mu\vec{v}$	Ratio of gravity force to viscous force	$L^2\rho g/\mu\vec{v}$	Re/Fr	Ratio of Reynolds number to Froude number
$PL/\mu\vec{v}^2$	Ratio of pressure force to viscous force	$PL/\mu\vec{v}$	Re·Eu	Product of Reynolds number and Euler number

condition has the same dimensions as those of the terms in the equation.

17.3.3.1 Mass-Transfer Equation

The mass transfer equation is given as

Rate of change of concentration

= Rate of change by convection + Rate of change by diffusion

+ Rate of change by homogenous chemical reaction

$$\frac{\partial c_A}{\partial t} = -\vec{v}\cdot\nabla c_A + D_{AB}\nabla^2 c_A + R_A. \tag{17.3}$$

Equation 17.3 can be expressed in terms of dimensional equality and it is given in Equation 17.4. Each term in Equation 17.4 has the dimension of $[N/L^3 t]$:

$$\left[\frac{c}{t}\right] = \left[\frac{\vec{v}c}{L}\right] + \left[\frac{D_{AB}c}{L^2}\right] + [R] \tag{17.4}$$

Likewise, there are six ratios that can be formed from the four terms in Equation 17.4.

$$\frac{L}{vt}, \frac{L^2}{D_{AB}t}, \frac{c}{Rt}, \frac{\vec{v}L}{D_{AB}}, \frac{\vec{v}c}{LR}, \frac{D_{AB}c}{L^2R}$$

Table 17.2 shows the dimensionless numbers obtained from the mass transfer equation. The dimensionless numbers also relate to some well-known dimensionless groups used in engineering.

Every term in the mass-transfer equation can be divided with mass-transfer coefficient, $(k_L c/L)$ that has the dimension of $[N/L^3 t]$. The following four dimensionless numbers are obtained.

$$\frac{L}{k_L t}, \frac{\vec{v}}{k_L}, \frac{D_{AB}}{k_L L}, \frac{LR}{k_L c}$$

Table 17.3 shows the additional dimensionless numbers obtained from dimensional analysis of mass transfer equation by division of every term of the mass transfer (or species) equation with the mass transfer coefficient.

Table 17.2 Dimensionless Numbers Obtained from Dimensional Analysis of the Mass Transfer Equation

Dimensionless Number in Equation 17.4	Formula	Symbol	Dimensionless Group
$L/\vec{v}t$	Reciprocal $= vt/L$	Th	Thomson number
$L^2/D_{AB}t$	Reciprocal $= D_{AB}t/L^2$	Fo_m	Fourier number
c/Rt	Reciprocal $= Rt/c$	$Da_I \cdot Th$	Product of Damkohler number and Thomson number
$\vec{v}L/D_{AB}$	vL/D_{AB}	Pe_m	Peclect number
$\vec{v}c/LR$	Reciprocal $= LR/vc$	Da_I	Damkohler number
$D_{AB}c/L^2R$	Reciprocal $= L^2R/D_{AB}c$	Da_{II}	Damkohler number

Table 17.3 Additional Dimensionless Numbers from the Mass Transfer Equation and Convective Boundary Condition

Dimensionless Number in Equation 17.4	Formula	Symbol	Dimensionless Group
$L/k_L t$	Reciprocal $= k_K t/L$	Fo$\cdot$*Sh*	Product of Fourier number and Sherwood number
$\vec{v}/k_L$	Reciprocal $= k_L/\vec{v}$	Sh/Pe_m	Ratio of Sherwood number to Peclet number
$D_{AB}/k_L L$	Reciprocal $= k_L L/D_{AB}$	Sh	Sherwood number
$LR/k_L c$	$LR/K_L c$	$Pe_m/Sh\cdot Da_1$	—

17.3.3.2 Energy Equation

The energy transfer equation is given as

Rate of change of temperature

= Rate of change by convection + Rate of change by conduction

+ Rate of generation by viscous dissipation + Rate of generation by chemical reaction

+ Rate of generation by Joule heating

$$\frac{\partial T}{\partial t} = -\vec{v}\cdot\nabla T + \frac{k}{\rho C_p}\nabla^2 T + \frac{\mu\phi}{\rho C_p} + \frac{QR_A}{\rho C_p} + \frac{I^2}{\sigma_e \rho C_p}. \tag{17.5}$$

The last term in Equation 17.5 that is the rate of generation by Joule heating is omitted in the dimensional analysis performed below. Equation 17.6 in the dimensional equality of Equation 17.5. Each term in Equation 17.6 has the dimension of $[T/t]$.

$$\left[\frac{T}{t}\right] = \left[\frac{\vec{v}T}{L}\right] + \left[\frac{kT}{\rho C_p L^2}\right] + \left[\frac{\mu\vec{v}^2}{\rho C_p L^2 :}\right] + \left[\frac{QR}{\rho C_p}\right]. \tag{17.6}$$

Here the boundary condition involving heat transfer coefficient ($hT/\rho C_p L$) is taken into account in the dimensional analysis. The division of terms in Equation 17.6 as well as the heat transfer coefficient yields the following dimensionless numbers:

$$\frac{L}{\vec{v}t}, \frac{\rho C_p L^2}{kt}, \frac{\rho C_p L^2 T}{\mu\vec{v}^2 t}, \frac{\rho C_p T}{QRt}, \frac{\rho C_p \vec{v} L}{k}, \frac{\rho C_p T}{\mu\vec{v}}, \frac{\rho C_p \vec{v} T}{LQR}, \frac{kT}{\mu\vec{v}^2}, \frac{kT}{L^2 QR}, \frac{\mu\vec{v}^2}{L^2 QR}, \frac{\rho C_p \vec{v}}{h}, \frac{\rho C_p \vec{v}}{hT}, \frac{k}{hL}, \frac{\mu\vec{v}^2}{LhT}, \frac{LQR}{hT}.$$

These dimensionless numbers include some well-known dimensionless groups that are frequently encountered in engineering; they are listed in Table 17.4.

Table 17.4 Dimensionless Numbers Obtained from Dimensional Analysis of the Differential Energy Conservation Equation

Dimensionless Number in Equation 17.4	Formula	Symbol	Dimensionless Group
$L/\vec{v}t$	Reciprocal $= \vec{v}t/L$	Th	Thomson number
$\rho C_P L^2/kt$	Reciprocal $= kt/\rho C_p L^2$	Fo	Fourier number
$\rho C_p L^2 T/\mu \vec{v}^2 t$	Reciprocal $= \mu \vec{v}^2 t/\rho C_p L^2 T$	Fo·Br	Product of Fourier number and Brinkman number
$\rho C_p T/QRt$	Reciprocal $= QRt/\rho C_p T$	Da_{III}·Th	Product of Damkohler number and Thomson number
$\rho C_p \vec{v} L/k$	$\rho C_p \vec{v} L/k$	Pe = Re·Pr	Product of Reynolds number and Prandtl number
$\rho C_p LT/\mu \vec{v}$	$\rho C_p LT/\mu \vec{v}$	Pe/Br	Ratio of Peclet number to Brinkman number
$\rho C_p \vec{v} T/LQR$	Reciprocal $= LQR/\rho C_p \vec{v} T$	Da_{III}	Damkohler number
$kT/\mu \vec{v}^2$	Reciprocal $= \mu \vec{v}^2/kT$	Br	Brinkman number
$kT/L^2 QR$	Reciprocal $= L^2 QR/kT$	Da_{IV}	Damkohler number
$\mu \vec{v}^2/L^2 QR$	$\mu \vec{v}^2/L^2 QR$	Br/Da_{IV}	Ratio of Brinkman number to Damkohler number
$\rho C_p L/ht$	Reciprocal $= ht/\rho C_p L$	Fo·*Nu*	Product of Fourier number and Nusselt number
$\rho C_p \vec{v}/h$	Reciprocal $= h/\rho C_p \vec{v}$	St = Nu/Pe	Stanton number
k/hL	Reciprocal $= hL/k$	Nu	Nusselt number
$\mu \vec{v}^2/LhT$	$\mu \vec{v}^2/LhT$	Br/Nu	Ratio of Brinkman number to Nusselt number
LQR/hT	LQR/hT	Da_{III}/St	Ratio of Damkohler number to Stanton number

17.3.3.3 Momentum-Transfer Equation

The momentum-transfer equation is given as

Rate of change of momentum

= Rate of change by convection + Rate of change by molecular viscous transfer

+ Rate of change due to pressure forces + Rate of change due to gravity forces

+ Rate of change due to magnetic forces

$$\frac{\partial \vec{v}}{\partial \mathrm{t}} = -\vec{v}\bullet\nabla\vec{v} + \frac{\mu}{\rho}\nabla^2\vec{v} - \frac{\nabla P}{\rho} + g\left(\mathrm{or} + g\frac{\Delta\rho}{\rho}\right) + \frac{\mu}{\rho}(I \times H_e). \tag{17.7}$$

Note that the left side term expresses acceleration that is the rate of change of momentum per unit mass of the fluid.

Likewise, the last term in Equation 17.7 that is the rate of change because of magnetic forces is omitted in dimensional analysis for the momentum-transfer equation. Expressed Equation 17.7 in term of dimensional equality, it yields Equation 17.8. Each term in Equation 17.8 has the dimension of $[L/t^2]$.

$$\left[\frac{\vec{v}}{t}\right] = \left[\frac{\vec{v}^2}{L}\right] + \left[\frac{\mu\vec{v}}{\rho L^2}\right] + \left[\frac{P}{\rho L}\right] + [g]\left(\mathrm{or}\left[g\frac{\Delta\rho}{\rho :}\right]\right). \tag{17.8}$$

Here, the sample boundary conditions involving wall shear stress, $(\tau/\rho L)$, as well as surface tension, $(\sigma/\rho L^2)$, are taken into account in the dimensional analysis. The division of terms in

Table 17.5 Dimensionless Numbers Obtained from Dimensional Analysis of the Differential Momentum Conservation Equation

Dimensionless Number in Equation 17.4	Formula	Symbol	Dimensionless Group
$L/\vec{v}t$	Reciprocal = $\vec{v}t/L$	Th	Thomson number
$\rho L^2/\mu t$	$\rho L^2/\mu t$	Re/Th	Ratio of Reynolds number to Thomson number
$\rho\vec{v}L/Pt$	Reciprocal = $Pt/\rho\vec{v}L$	Eu·Th	Product of Euler number and Thomson number
$\vec{v}/gt$	$\vec{v}/gt$	Fr/Th	Ratio of Froude number to Thomson number
$\rho\vec{v}/gt\Delta\rho$	$\rho\vec{v}/gt\Delta\rho$	$Re^2/Gr\cdot Th$	—
$\rho\vec{v}L/\mu$	$\rho\vec{v}L/\mu$	Re	Reynolds number
$\rho\vec{v}^2/P$	Reciprocal = $P/\rho\vec{v}^2$	Eu	Euler number
$\vec{v}^2/gL$	$\vec{v}^2/gL$	Fr	Froude number
$\rho\vec{v}^2/gL\Delta\rho$	$\rho\vec{v}^2/gL\Delta\rho$	Re/Gr	Ratio of Reynolds number square to Grash of number
$\mu\vec{v}/PL$	Reciprocal = $PL/\mu\vec{v}$	Re·Eu	Product of Reynolds number and Euler number
$\mu\vec{v}/\rho gL^2$	$\mu\vec{v}/\rho gL^2$	Fr/Re	Ratio of Froude number to Reynolds number
$\mu\vec{v}/gL^2\Delta\rho$	$\mu\vec{v}/gL^2\Delta\rho$	Re/Fr	Ratio of Reynolds number to Froude number
$P/\rho gL$	Reciprocal = $\rho gL/P$	Eu·Fr	Product of Euler number and Froude number
$P/gL\Delta\rho$	$P/gL\Delta\rho$	$Eu\cdot Re^2/GR$	—
$\rho/\Delta\rho$	—	—	—
$\rho\vec{v}L/t\tau$	Reciprocal = $t\tau/\rho\vec{v}L$	Ne·Th	Product of Newton number and Thomson number
$\rho\vec{v}L^2/t\sigma$	$\rho\vec{v}L^2/t\sigma$	We/Th	Ratio of Weber number to Thomson number
$\rho\vec{v}^2/\tau$	Reciprocal = $\tau/\rho\vec{v}^2$	Ne	Newton number
$\rho\vec{v}^2L/\sigma$	$\rho\vec{v}^2L/\sigma$	We	Weber number
$\mu\vec{v}/L\tau$	Reciprocal = $L\tau/\mu\vec{v}$	Re·Ne	Product of Reynolds number and Newton number
$\mu\vec{v}/\sigma$	$\mu\vec{v}/\sigma$	Ca = We/Re	Capillary number
P/τ	P/τ	We/Ne	Ratio of Weber number to Newton number
PL/σ	PL/σ	Eu/We	Ratio of Euler number to Weber number
$\rho gL/\tau$	Reciprocal = $\tau/\rho gL$	Ne·Fr	Product of Newton number and Froude number
$\rho gL^2/\sigma$	$\rho gL^2/\sigma$	We/Fr	Ratio of Weber number to Froude number

Equation 17.8 as well as the two coefficients yield the following dimensionless numbers:

$$\frac{L}{\vec{v}t},\ \frac{\rho L^2}{\mu t},\ \frac{\rho\vec{v}L}{Pt},\ \frac{\vec{v}}{gt},\ \frac{\rho\vec{v}}{gt\Delta\rho},\ \frac{\rho\vec{v}L}{\mu},\ \frac{\rho\vec{v}^2}{P},\ \frac{\vec{v}^2}{gL},\ \frac{\rho\vec{v}^2}{gL\Delta\rho},\ \frac{\mu\vec{v}}{PL},\ \frac{\rho C_p\vec{v}}{h},\ \frac{\rho C_p\vec{v}}{hT},\ \frac{P}{\rho gL},$$
$$\frac{P}{gL\Delta\rho},\ \frac{\rho}{\Delta\rho},\ \frac{\rho\vec{v}L}{t\tau},\ \frac{\rho\vec{v}L^2}{t\sigma},\ \frac{\rho\vec{v}^2}{\tau},\ \frac{\rho v^2L}{\sigma},\ \frac{\mu\vec{v}}{L\tau},\ \frac{\mu\vec{v}}{\sigma},\ \frac{P}{\tau},\ \frac{PL}{\sigma},\ \frac{\rho gL}{\tau},\ \frac{\rho gL^2}{\sigma},\ \frac{gL\Delta\rho}{\tau},$$
$$\frac{gL^2\Delta\rho}{\sigma}.$$

Table 17.5 lists the dimensionless numbers shown above, and these numbers can be related to some well-known dimensionless groups frequently used in engineering.

17.4 LIST OF DIMENSIONLESS NUMBERS

A list of dimensionless numbers that are frequently used in modeling or correlating mass, heat, and momentum transport processes as well as coupled processes is given in Table 17.6. Formulas and interpretation of the dimensionless numbers are given as well as the areas where these numbers are applied.

Table 17.6 Dimensionless Numbers Encountered in the Literature Dealing with Transport Processes

Group	Formula	Interpretation	Application
Archimedes number	$Ar = gL^3\rho_f(\rho - \rho_f)/\mu^2$ g-gravitational acceleration (m/s^2) L—characteristic length (m) ρ—density of body (kg/m^3) ρ_f—density of fluid (kg/m^3) μ—viscosity (kg/s m)	Ratio of gravitational forces to viscous force, used to relate motion of fluids and particles due to density differences, as for fluidized beds	Momentum transfer (general); buoyancy, fluidization, and fluid motion due to density difference
Biot number	$Bi = (hL/k_s)$ h—overall heat transfer coefficient (W/m^2 · K) L—characteristic length (m) k_s—thermal conductivity of solid (W/m · K)	Ratio of the internal thermal resistance of a solid to the boundary layer (or surface film) thermal resistance Can also be regarded as ratio of conductive to convective heat resistance Biot number relates the heat transfer resistance inside and the surface of a solid $Bi > 1$ implies that the heat conduction inside the solid is slower than at its surface. Thus, temperature gradient inside the solid cannot be neglected	Unsteady state heat transfer
Mass transfer Biot number	$Bi_m = h_m L/D_{AB}$ h_m—overall mass transfer coefficient (m/s) L—characteristic length (m) D_{AB}—binary mass diffusion coefficient (m/s^2)	Ratio of mass transfer resistance in internal species to mass transfer resistance at boundary layer (interface) species Can also be regarded as ratio of diffusive to convective mass transfer resistance	Mass transfer between fluid and solid
Boltzmann number	$Bo = (\vec{v} c_p(sw))/\eta T$ $\vec{v}$— velocity (m/s) c_p—specific heat at constant pressure (J/kg · K) sw-specific weight (N/m^3) η—Stefan-Boltzmann constant T—absolute temperature (K)	Boltzmann number is a parameter of thermal radiation exchange that relates the enthalpy of gases and heat flow emitted at the surface	Simultaneous heat and momentum transfer
Bond number	$Bo = g(\rho_1 - \rho_v)L^2)/\sigma$ g—gravitational acceleration (m/s^2) ρ—density (kg/m^3) L—characteristic length (m) σ—surface tension, undisturbed surface tension (N/m)	Ratio of gravitational forces to surface tension forces	Momentum transfer (general); atomization, motion of bubbles and droplets

continued

Table 17.6 continued

Group	Formula	Interpretation	Application
Brinkman number	$Br = \mu\vec{v}^2/k\Delta T$ μ—viscosity (kg/s·m) $\vec{v}$—fluid velocity; local velocity (m/s) K—thermal conductivity (W/m·K) ΔT—temperature difference	Ratio of heat production by viscous dissipation to the heat transport by conduction	Heat transfer
Bulygin number	$Bu = (\lambda c_v P(T - T_0)/c)$ λ—latent heat of phase change (kJ/kg) c_v—specific vapor capacity (kJ/kg °C) P—pressure, local static pressure (Pa) T—temperature of medium/moist surface (wet-bulb temperature)/moving stream T_0—initial temperature/hot gas stream c—heat capacity of moist material (kJ/kg °C)	Ratio of heat of vaporization to sensible heat to bring liquid to boiling point Bulygin number represents high intensity heat and mass transfer during evaporation	Heat transfer during evaporation
Capillary number	$Ca = \mu\vec{v}/\sigma$; $Ca = We/Re$ μ—viscosity (kg/s m) $\vec{v}$—fluid velocity; local velocity (m/s) σ—surface tension, undisturbed surface tension (N/m)	Ratio of viscous forces to surface tension forces Capillary number describes the flow of fluids through thin tubes (capillaries)	Momentum transfer (general); atomization and two-phase flow in beds of solids
Coefficient of friction	$C_f = \tau_s/(\rho\vec{v}/2)$ τ_s—shear stress (N/m^2) ρ—density (kg/m^3) $\vec{v}$—velocity (m/s)	Ratio of the force that maintains contact between an object and a surface and the frictional force that resists the motion of the object Can also be regarded as dimensionless surface shear stress	Momentum transfer
Dean number	$De = (d\vec{v}\rho/\mu)\sqrt{r/r_{eff}}$ d—diameter of pipe, particle, bubble, droplet, impeller, shaft, etc., (m) $\vec{v}$—velocity (m/s) ρ—density (kg/m^3) μ—viscosity (kg/s m) r—radius of pipe, particle, bubble, droplet, impeller, shaft, etc., (m) r_{eff}—radius of curvature of bend	Ratio of centrifugal force to inertia force in fluid flow in a curve duct	Momentum transfer (general); flow in curved channels

Deborah number	$De = t_r/t_0$ t_r—relaxation time or reaction time (s) t_0—observation time (s) $De = (t_r D_{AB}/L^2)$ t_r—relaxation time or reaction time (s) D_{AB}—binary mass diffusion coefficient (m/s^2) L—characteristic length (m)	The first equation shows the ratio of relaxation time to observation time. The second equation shows the ratio of the relaxation time to the characteristics diffusion time Deborah number indicates how "fluid" a material is; the smaller the Deborah number, the more fluid the material appears	Rheology
Eckert number	$Ec = \vec{v}^2/(c_p(T_s - T_\infty))$ $\vec{v}$—velocity (m/s) c_p—specific heat at constant pressure (J/kg·K) T—temperature (K)	Ratio of kinetic energy of the flow to the boundary layer enthalpy difference. Can also be regarded as ratio of temperature rise to temperature difference between wall and fluid at the boundary layer in adiabatic flow	Momentum and heat transfer (general); compressible flow
Euler number	$Eu = \Delta P/\rho\vec{v}^2$ ΔP—pressure drop due to friction (Pa) ρ—density (kg/m^3) $\vec{v}$—velocity (m/s)	Ratio of friction head to two velocity head Euler number represents the relationship between the pressure drop due to friction and the inertia forces in a flow stream	Momentum transfer (general); fluid friction in conduits
Fedorov number	$Fe = d_e\{(4g\rho^2(\rho_p/\rho_g))/3\mu^2\}^{1/3}$ d_e—equivalent particle diameter (m) g—gravitational acceleration (m/s^2) ρ—density (kg/m^3) ρ_p—density of particle or droplet (kg/m^3) ρ_g—density of gas (kg/m^3) μ—viscosity (kg/s m)	Fedorov number represents the relationship between the flow of particles and gas in a fluidized bed	Fluidized beds
Fourier number	$Fo = \alpha t/L^2$; $\alpha = k/c_p\rho$ α—thermal diffusivity (m^2/s) t—time (s) L—characteristic length (m)	Ratio of the heat conduction rate to the rate of thermal energy storage in a solid Can also be regarded as ratio of current time to time to reach steady state Dimensionless time in temperature curves, used in explicit finite difference stability criterion Fourier number characterizes the connection between the rate of change of temperature, physical properties, and the dimension of the product in the unsteady heat transfer, conduction for heat transfer	Heat transfer (general); unsteady state heat transfer
Mass transfer Fourier number	$Fo_m = D_{AB}t/L^2$ D_{AB}—binary mass diffusion coefficient (m/s^2) t—time (s)	Ratio of the species diffusion rate to the rate of species storage Dimensionless time	Mass transfer (general); unsteady state mass transfer

continued

Table 17.6 continued

Group	Formula	Interpretation	Application
	L—characteristic length (m)	It characterizes the connection between the rate of change of temperature, physical properties, and the dimension of the product in the unsteady mass transfer, diffusion for mass transfer	
Friction factor	$f=\Delta P/((L/D)(\rho \vec{v}_f^2/2))$ ΔP—pressure drop (N/m^2) L—characteristic length (m) D—diameter (m) ρ—density (kg/m^3) $\vec{v}_f$—mass average fluid velocity (m/s)	Friction factor expresses the linear relationship between mean flow velocity and pressure gradient Can also be regarded as dimensionless pressure drop for internal flow	Internal flow (general); fluid friction in conduits
Froude number	$Fr=\vec{v}/gL$ $\vec{v}$—velocity (m/s) g—gravitational acceleration (m/s^2) L—characteristic length (m)	Ratio of inertia force and gravity force in homogenous fluid flow In fluid dynamics, the Froude number is the reciprocal of the square root of the Richardson number	Momentum transfer (general); open channel flow and wave and surface behavior
Galileo number	$Ga=L^3 g\rho^2/\mu^2$ L—characteristic length (m) g—gravitational acceleration (m/s^2) ρ—density (kg/m^3) μ—viscosity (kg/s·m)	Ratio of gravity force to viscous force Galileo number measures the force of molecular friction and the force of gravity in fluid flow, particularly for a viscous fluid	Momentum and heat transfer (general); viscous flow/circulation and thermal expansion calculations in particular
Gay Lussac number	$Ga=1/\beta\Delta T$ β—coefficient of bulk expansion (K^{-1}) ΔT—liquid superheat temperature difference (K)		Thermal expansion processes
Graetz number	$Gz=\dot{m}c_p/kL$ $\dot{m}$—mass flow rate (kg/s) c_p—specific heat at constant pressure (kJ/kg·°C) k—thermal conductivity (W/m·K) L—characteristic length (m)	Ratio of thermal capacity fluid to convective heat transfer in forced convection of a fluid in streamline flow Equivalent to $\{(L/d)/(Re\cdot Pr)\}$ or $\{(L/d)/Pe\}$	Heat transfer (general); streamline flow, convection in laminar flow
Grashof number	$Gr=(g\beta(T_s-T_\infty)L^3)/v^2$; $v=\mu/\rho$ g—gravitational acceleration (m/s^2) β—volumetric thermal expansion coefficient (K^{-1}) T—temperature (K) L—characteristic length (m) v—kinetic viscosity (m^2/s)	Ratio of natural convection buoyancy force to viscous force acting on fluid in natural convection	Heat transfer (general); free convection

Colburn j factor (heat transfer)	$j_H = St \cdot Pr^{2/3}$ $j_H = (h/c_p\rho v)(c_p\mu/k)^{2/3}$ St—Stanton number Pr—Prandtl number	Dimensionless heat transfer coefficient	Heat transfer (general); free and forced convection
Colburn j factor (mass transfer)	$j_m = St_m Sc^{2/3}$ St_m—mass transfer Stanton number Sc—Schmidt number	Dimensionless mass transfer coefficient	Mass transfer (general)
Jakob number	$Ja = c_p(T_s - T_{sat})/h_{fg}$ c_p—specific heat at constant pressure (J/kg·K) T—temperature (K) λ—latent heat of vaporization (J/kg)	Ratio of sensible to latent heat energy absorbed during liquid-vapor phase change	Heat transfer (general); liquid-vapor phase change
Karman number	$Ka = (\rho d^3(-dP/dL))/\mu^2$ ρ—density (kg/m^3) d—diameter of pipe, particle, bubble, droplet, impeller, shaft, etc., (m) P—pressure, local static pressure (Pa) L—characteristic length (m) μ—viscosity (kg/s·m)	Karmen number is a measure of stream turbulence in fluid flow	Momentum transfer (general); fluid friction in conduits
Kirpichev number (heat transfer)	$Ki_h = qL/K\Delta T$ q—heat flux (W/m^2) L—characteristic length (m) k—thermal conductivity (W/m·K) ΔT—liquid superheat temperature difference (K)	Ratio of external heat transfer intensity to internal heat transfer intensity Kirpichev number relates the drying conditions for heat transfer	Heat transfer (general)
Kirpichev number (mass transfer)	$Ki_m = GL/D_{AB}\rho n$ G—mass low rate per unit area (kg/m^2·s) L—characteristic length (m) D_{AB}—Binary mass diffusion coefficient (m^2/s) ρ—density (kg/m^3) n—specific mass constant (kg/kg)	Ratio of external mass transfer intensity to internal mass transfer intensity Kirpichev number relates the drying conditions for mass transfer	Mass transfer (general)
Knudsen number	$Kn = (L_{mfp}/L)$ L_{mfp}—length of mean free path (m) L—characteristic length (m)	Ratio of the molecular mean free path length to characteristics physical length Knudsen number represents the relationship of the mean free path of gaseous diffusion in drying beds in relation to the characteristic length dimension of the bed	Momentum and mass transfer (general); very low pressure gas flow
Kossovich number	$Ko = \lambda X/c_p\Delta T$ λ—latent heat of phase change (kJ/kg) X—moisture content (kg/kg)	Ratio of heat used for evaporation to heat used in raising temperature of body	Heat transfer (general); convective heat transfer during evaporation

continued

Table 17.6 continued

Group	Formula	Interpretation	Application
	c_p—specific heat (kJ/°C kg) ΔT -liquid superheat temperature difference (K)		
Lebedov number	$Le=(\varepsilon b_t(T_{surr}-T_0))/c_v P \rho_s$ ε—voids b_t—vapor expansion in capillaries (kg/m^{3}K) T_{surr}—temperature of surrounding medium (K) T_0—initial temperature (K) c_v—specific vapor capacity (kg/kg Pa) P—pressure (Pa) ρ_s—density of solids (kg/m^3)	Ratio of the molar expansion flux to the molar vapor transfer flux for drying of porous materials	Drying of porous materials
Lewis number	$Le=\alpha/D_{AB}$; $\alpha=k/c_p\rho$; $Le=Sc/Pr$ α—thermal diffusivity (m^2/s) D_{AB}—binary mass diffusion coefficient (m/s^2)	Ratio of the thermal diffusivity and mass diffusivity of a material Also known as the Lykov–Lewis number	Combined heat and mass transfer
Miniovich number	$Mn=SR/\varepsilon$ S: particle area/particle volume (m^{-1}) R: radius of pipe, pore, shaft, etc.; radius of curvature of bend (m) ε: voidage or porosity [−]	Miniovich number relates the pore size and porosity of a product being dried	Drying
Newton number	$Ne=\tau/\rho\vec{v}^2$ τ—torque or shear stress ρ—density (kg/m^3) $\vec{v}$—fluid velocity (m/s)	Ratio of drag force to inertia force	Momentum transfer
Nusselt number	$Ne=\tau/\rho\vec{v}^2$ h—overall heat transfer coefficient (W/m$^2\cdot$K) L—characteristic length (m) k_f—thermal conductivity of fluid (W/m·K)	Ratio of the total heat transfer to the conduction heat transfer in forced convection Dimensionless temperature gradient at the surface It is used to calculate heat transfer coefficient, h	Heat transfer (general); forced convection
Ostrogradsky number	$Os=q_v L^2/k\Delta T$ q_v—strength of the internal heat source (W/m^3) L—characteristic length (m) k—coefficient of thermal conductivity of medium (W/m K) ΔT—temperature difference (K)	Ostrogradsky number relates the internal heating of a product the thermal properties of the medium (solid, liquid, gas)	Heat transfer
Peclet number	$Pe=\vec{v}L/\alpha$; $\alpha=k/c_p\rho$; $Re\cdot Pr$ $\vec{v}$—fluid velocity (m/s)	Ratio of bulk heat transfer to conductive heat transfer	Heat transfer (general); forced convection

	L—characteristic length (m) α—thermal diffusivity (m^2/s)	It is a dimensionless independent heat transfer parameter	
Peclet number (mass transfer)	$Pe_m = vL/D_{AB}$; $Pe_m = Re\ Sc$ $\vec{v}$—fluid velocity (m/s) L—characteristic length (m) D_{AB}—binary mass diffusion coefficient (m/s^2)	Ratio of convective mass transfer to diffusive mass transfer It is a dimensionless independent mass transfer parameter	Mass transfer
Prandtl number	$Pr = c_p\mu/k$); $Pr = (v/\alpha)$ c_p—specific heat at constant pressure (J/kg K) μ—viscosity (kg/s m) k—thermal conductivity (W/m·K) v—kinetic viscosity (m^2/s)	Ratio of momentum diffusivity and thermal diffusivity $Pr = 1$ gives boundary layers of equal thickness $Pr > 1$ gives a thinner velocity boundary layer as momentum transfer is more rapid than heat transfer	Heat transfer (general); free and forced convection
Rayleigh number	$Ra = L^3\rho g\beta\Delta T/\mu\alpha$ $\alpha = k/c_p\rho$ L—characteristic length (m) ρ—density (kg/m^3) g—gravitational acceleration (m/s^2) β—volumetric thermal expansion coefficient (K^{-1}) T—temperature (K) μ—viscosity (kg/s·m) α—thermal diffusivity (m^2/s)	Ratio of natural convective to diffusive heat/mass transport	Heat transfer (general); free convection
Reynolds number	$Re = \vec{v}L/v$; $Re = \rho\vec{v}L/\mu$ $\vec{v}$—velocity (m/s) L—characteristic length (m) V—kinetic viscosity (m^2/s)	Ratio of inertia force and viscous force This number provides a criterion for determining dynamic similarity	Momentum, heat, and mass transfer to account for dynamic similarity
Schmidt number (mass transfer Prandtl number)	$Sc = v/D_{AB}$; $v = (\mu/\rho)$ v—kinetic viscosity (m^2/s) D_{AB}—binary mass diffusion coefficient (m/s^2)	Ratio of the momentum and mass diffusivities	Mass transfer (general); diffusion in flowing systems
Sherwood number (mass transfer Nusselt number)	$Sh = h_m L/D_{AB}$; $Sh = j_m \cdot Re \cdot Sc^{1/3}$ h_m—overall mass transfer coefficient (m/s) L—characteristic length (m) D_{AB}—binary mass diffusion coefficient (m/s^2)	Ratio of length scale to the diffusive boundary layer thickness Sherwood number represents dimensionless concentration gradient at the surface; it is used to calculate mass transfer coefficient h_m	Mass transfer
Stanton number	$St = h/\rho\vec{v}c_p$; $St = Nu/Re \cdot Pr$ h—overall heat transfer coefficient (W/m^2 K) ρ—density (kg/m^3) $\vec{v}$—velocity (m/s) c_p—specific heat at constant pressure (J/kg K)	Ratio of heat transfer to momentum transfer Stanton number is a modified Nusselt number	Heat transfer (general); forced convection

continued

Table 17.6 continued

Group	Formula	Interpretation	Application
Mass transfer Stanton number	$St_m = h_m/\vec{v}$; $St_m = Sh/Re \cdot Sc$ h_m—overall mass transfer coefficient (m/s) $\vec{v}$—velocity (m/s)	Ratio of mass transfer to momentum transfer; it is a modified Sherwood number	Mass transfer (general); forced convection
Thomson number	$Th = \vec{v}t/L$ $\vec{v}$—velocity (m/s) L—characteristic length (m) t—time (s)	Ratio of convective transport to storage of quantity in question In mass transfer, it is the ratio of the rate of convective mass transfer to the rate of species storage In heat transfer, it is the ratio of the rate of bulk heat transfer (convective) to the rate of thermal energy storage In momentum transfer, it is the ratio of inertia force to species rate of change of velocity	Mass, heat and momentum transfer
Weber number	$We = \rho\vec{v}^2L/\sigma$ ρ—density (kg/m^3) $\vec{v}$—velocity (m/s) L—characteristic length (m) η—Stefan–Boltzmann constant	Ratio of inertia force to surface tension force	Momentum transfer (general), bubble/droplet formation, and breakage of liquid jets

This list is rather comprehensive and includes some groups found only in the Russian literature.
Source: From Hall, C. W., *Drying Technology*, 10(4), 1081–1095, 1992.

Table 17.7 Category of Dimensionless Numbers

Transport Processes	Dimensionless Numbers
Momentum transfer	Ar, Bo (Bond), Ca, C_f, De (Dean), Eu, f, Fr, Ga, Ka, Re, We
Mass transfer	Bi_m, Fo_m, j_m, Ki_m, Pe_m, Re, Sc, Sh, St_m
Heat transfer	Bi, Bu, Fo, Gz, Gr_L, J_H, Ja, Ki_h, Ko, Nu, Os, Pe_L, Pr, Ra, Re, St
Simultaneous heat and mass transfer	Bo (Boltzmann), Ec, Ga
Simultaneous mass and momentum transfer	Kn
Simultaneous heat and momentum transfer	Le, Lu
Specific area	
Fluidization	Ar, Fe
Drying	Kn, Le, Mn; Bi,[8,9] Di(Dincer)[8]
Atomization, bubbles, and droplets	Bo (Bond), Ca, We
Evaporation	Bu, Ja, Ko
Rheology	De (Deborah)
Compressible fluid	Ec
Thermal expansion	Ga
Fluid motion	Ar
Pneumatic transport (coarse grain materials)	Eu, f, Fr, Re[10]

The dimensionless numbers listed in Table 17.6 are categorized into three main transport processes, viz. heat, mass, and momentum transfers, and they are listed in Table 17.7. Dimensionless numbers that are frequently used in specific transport processes such as fluidization, drying, evaporation, and etc., are also listed in Table 17.7.

Table 17.8 lists some illustrative research findings on the application of dimensional analysis and the respective π space that consists of the dimensionless numbers of the process. Most textbooks of fluid mechanics, heat, or mass transfer provide detailed analysis of dimensional analysis and its application.

17.5 GENERALIZATION OF EXPERIMENTAL DATA TO OBTAIN EMPIRICAL CORRELATIONS

After the dimensionless numbers associated with a particular process in question are identified, experiments are designed and performed; the experiments are repeated for different values of the variables listed in the relevance list. Then the experimental result is analyzed and plotted on an $x-y$ plot. Analysis of the experimental results will show that certain variables have negligible influence and are irrelevant to the process in question. Such variables should then be removed form the relevance list and the dimensional analysis repeated. Including irrelevant parameters can distort the usefulness of dimensional analysis.

If experimental data is plotted on the graph lie and concentrated in a small band, then it indicates that an equation or an expression can be used to represent the relationship of the variables plotted on the graph. The equation obtained from the plot sometimes is valid only over the range of conditions experimentally tested. The function that formed the equation may consist of a dimensionless number or a function of several dimensionless numbers and parameters.

17.6 APPLICATIONS OF DIMENSIONAL ANALYSIS

Dimensional analysis has been applied to many areas, including areas related to food processing. In addition, many empirical equations and correlations that relate different dimensionless numbers in the respective π-space have been obtained. Here two examples are given to show in detail how the correlations and relationships between dimensionless groups are obtained from experimental findings.

Table 17.8 Some Research Findings on the Application of Dimensional Analysis in Areas Related to Food Processing

Operation	Relevance List {Target Quantity; Geometric Parameters; Material Properties; Process Parameters}	π-Space (References)
Mixing	{variation coefficient (mixing quality), χ; Drum diameter (D), drum length (L), mixing devise diameter (d) mean particle diameter (d_p), degree of fill of drum (ϕ); Effective axial dispersion coefficient (D_{eff}), particle density (ρ); Mixer rotational speed ($\vec{v}_\theta$), mixing time (θ), solid gravity ($g\rho$)}	Zlokarnik:[2] {χ, L/D, d/D, d_p/D, ϕ, $\theta\vec{v}_\theta$, Bo, Fr}
Drying	{Moisture ratio (MR); Film thickness (δ), film length (L); Gas density (ρ_g), solvent density (ρ_L), gas kinematic viscosity (v), gas heat capacity (C_{pg}), solvent heat capacity (C_{pl}), gas thermal diffusivity (α), solvent diffusivity (D_{AB}), solvent mass transfer coefficient (k_L), solvent vapor pressure (P_{vL}), Solvent heat of evaporation (ΔH); Gas throughput (F), gas pressure (P), gas temperature (T), drying time (t)}	Zlokarnik:[2]{MR; δ/L; ρ_L/ρ_g, C_{pL}/C_{pg}, P_{vL}/P, Sc, Sh, $k_L^2/\Delta H$, Pr; Fo, $\Delta H/C_{pL}T$, $P/\rho\Delta H$, Re}
Bubbling gas fluidized bed	{Target quantity; Bed diameter (D), Bed height (H) Fluid density (ρ_f), solid density (ρ_s), particle sphericity (ψ), fluid viscosity (μ), particle diameter (d) Superficial gas velocity (u_0), gravity acceleration (g)}	Zhang and Yang:[14] {gD/u_0^2, $\rho_s^2 g(\psi d)^4/\mu_f^2 D$} for $Re_p \leq 4$ {gD/u_0^2, $\rho_f D/\rho_s \psi_s d$} for $Re_p \geq 400$ {gD/u_0^2, $\rho_s^2 g(\psi d)^4/\mu_f^2 D$, $\rho_f D/\rho_s \psi_s d$} for $4 \leq Re_p \leq 400$
Circulating fluidized bed	{Target quantity; Column diameter (D); Particle diameter (d_p), particle density (ρ_p), gas density (ρ_g), gas viscosity (μ); External solids circulation flux (G_s), superficial gas velocity ($\vec{v}_g$), acceleration gravity (g)}	van der Meer, Thorpe, and Davidson:[15] d/D, ρ_p/ρ_g, $G_s/\rho_p u_0$, Re, Fr
Spouted bed	{Target quantity; Column diameter (D), bed height (H); Particle diameter (d_p), particle density (ρ_p), gas density (ρ_g), gas viscosity (μ), particle sphericity (ψ), bed voidage (ε), internal friction angle (ϕ); Superficial gas velocity (u_0), acceleration gravity (g)}	He, Lim, and Grace:[16] gd_p/u^2, $\rho_p d_p u_o/\mu$, ρ_g/ρ_p, H/d_p, D/d_p, ψ, ε, ϕ

17.6.1 Convective Heat-Transfer Coefficients in Cans

Thermal processing or canning is one of the most effective methods of food preservation and assurance of bacteriological safety. Since the early 1950s, agitation sterilization processing has been recognized as an effective method for achieving high quality foods. During agitation processing, the heat transfer to particulate liquids in cans is considerably more complex. Examples of particulate liquid cans are vegetable chunks in brine, fruit pieces in syrup or juices, meatballs in tomato sauce, etc. To establish a thermal processing schedule for such systems, experimental transient temperatures of liquid and particle center are needed. Theoretical models can also be used for the design, optimization, and validation of such systems. Overall heat transfer coefficient from heating medium to canned liquid (U) and liquid to particle heat transfer coefficient (h_{fp}) data can be used for the prediction of temperature profiles for liquid and solid particles besides relevant thermal and physical properties. The convective heat transfer coefficients associated with canned foods undergoing thermal processing are influenced by various operating conditions as well as liquid and particle properties.[11,12]

The convective heat-transfer coefficients (U and h_{fp}) are expressed in terms of the Nusselt number (Nu) that is a function of other dimensionless numbers, consisting of relevant properties of the liquid, particles, and system. Sablani et al.[12] developed dimensionless correlations for estimating convective heat transfer coefficients for cans with rotational processing. They used the data of experimental and mathematical study conducted by Sablani.[11] A summary of the range of operating and product parameters used in the determination of convective heat transfer coefficients is presented in Table 17.9. The experimental and mathematical procedure for estimation of the convective heat transfer coefficients in cans is described in Sablani et al.[12]

Experimental data obtained for U and h_{fp} were used to calculate the Nusselt number using the relationship $Nu = U$ (of h_{fp}) d_{cd}/k_l where d_{cd} and k_l are the characteristic dimension and thermal conductivity of the liquid, respectively. Other dimensionless numbers were calculated using the physical properties of liquid and particle (at average bulk temperature) and system (operating) parameters. The characteristic length $D_r + D_c$ (diameter of rotation + diameter of can) was used in the Nu based on overall heat-transfer coefficient, and equivalent particle diameter ($d_e = (6\times$ volume of particle$/\pi)^{0.33}$) was used in the Nu based on liquid to particle heat transfer coefficient.[12]

Analysis of variance on the experimental data has shown that the convective heat transfer coefficients (U and h_{fp}) are influenced by rotational speed, liquid viscosity, particle size, shape, and concentration. Therefore, the Nusselt number (Nu) was modeled as a function of relevant dimensionless groups

Overall heat-transfer coefficient (U):

$$Nu = f\left(\text{Re}, \text{Pr}, Fr, Ar, \frac{\varepsilon}{100-\varepsilon}, \frac{D_e}{D_c}, \psi\right). \quad (17.9)$$

Liquid-to-particle heat-transfer coefficient (h_{fp}):

$$Nu = f\left(\text{Re}, \text{Pr}, \frac{k_p}{k_l}, \frac{\varepsilon}{100-\varepsilon}, \frac{D_e}{D_c}, \psi\right). \quad (17.10)$$

For the overall heat-transfer coefficient, a stepwise multiple regression analysis of experimental data on various factors, represented in dimensionless form, eliminated Froude number as nonsignificant ($P > 0.05$) in comparison with the other parameters. The following equation gave the best fit

Table 17.9 System and Product Parameters Used in the Determination of Convective Heat-Transfer Coefficients

System and Product Parameters	Symbol	Experimental Range
Heating medium temperature	T_R	110, 120, and 130°C
Diameter of rotation	D_r	0, 0.18, 0.38, and 0.54 m
Rotation speed	N	10, 15, and 20 rpm
Can liquids	Water and oil	
Particle type and shape	Nylon and sphere diameter (D)	0.01905, 0.02225, and 0.025 m
	Cube (L_{cu})	0.01905 m
	Cylinder ($L_{cyl} \times D_{cyl}$)	0.01905 × 0.01905 m
Particle concentration	ε	20, 30, and 40% (v/v)

($R^2 = 0.99$) for the experimental data for the overall heat transfer coefficient with multiple particles:

$$Nu = 0.71\text{Re}^{0.44}\text{Pr}^{0.36}\left(\frac{\varepsilon}{100-\varepsilon}\right)^{-0.37}\left(\frac{D_e}{D_c}\right)^{-0.11}\psi^{0.24}. \quad (17.11)$$

The above correlation (Equation 17.11) is valid for the Re in the range 1.7×10^4–5.4×10^5, Pr in the range of 2.6–90.7, ε in the range of 20–40% (v/v), the ratio of d_e/D_c in the range of 0.22–0.29, and ψ in the range of 0.806–1.

Regression analysis of the experimental data, obtained for liquid to particle heat transfer coefficient in the presence of multiple particles, gave the following correlation ($R^2 = 0.96$):

$$Nu = 0.167\text{Re}^{0.61}\left(\frac{k_p}{k_l}\right)^{1.98}\left(\frac{\varepsilon}{100-\varepsilon}\right)^{0067}\left(\frac{D_e}{D_c}\right)^{-0.70}\psi^{0.23} \quad (17.12)$$

The correlation (Equation 17.12) is valid for Re in the range of 28–1.55×10^3, k_p/k_l in the range from 0.56 to 2.24, d_e/D_c in the range from 0.22 to 0.29, ε in the range from 20 to 40% (v/v), and ψ in the range from 0.806 to 1.

17.6.2 Fastest Particle Flow in an Aseptic Processing System

In aseptic processing, the food is first heated in scraped surface heat exchangers (SSHE) and held for a pre-determined time in a hold tube, cooled quickly through a second set of SSHEs, filled, and aseptically sealed into sterile containers. Residence time distribution of particles and liquid to particle heat transfer coefficient are needed for process calculations. Residence time distribution of particles is critical because different particles take varying amounts of time to pass through the holding tube. The residence time of the fastest particle is required from the process safety point of view. Knowledge of the flow characteristics of viscous liquid and suspended food particles in SSHEs and holding tubes is essential to continuous aseptic processing of low acid liquid foods containing particulates.[13] Abdelrahim et al.[13] developed dimensionless correlations to describe the flow behavior of food particles (meat and carrot cubes) in the SSHE, holding tube, and the entire assembly of a pilot scale aseptic processing system.

Experimental study involved the determining of residence time of the fastest particle (meat or carrot) in holding tubes and the whole SSHE. The carrier fluid was starch solution of different concentration. The experimental conditions are listed in the Table 17.10. The details of the aseptic processing system and properties of carrier liquid and particles are described in Abdelrahim et al.[13]

Table 17.10 Different Experimental Conditions Used in the Particle Residence Time Study in the Aseptic Processing System

Parameters	Range
Carrier fluid	Thermo-flo starch (gelatinization temperature 140°C)
Concentration of starch	3, 4, 5, 6%
Density	1010, 1014, 1019, 1026 kg/m^3
Flow rate	10, 15, 20, 25 kg/min
Particles	Carrots and meat
Particle size (carrot)	0.007, 0.016 m
Particle size (meat)	0.012, 0.019, 0.025 m
Particle concentration (carrot)	5%
Particle concentration (meat)	5%
Particle density (carrot)	1040 kg/m^3
Particle density (meat)	1110 kg/m^3

A particle moving in viscous liquid experiences a change of momentum equal to the sum of imposed forces: gravitational, buoyancy, drag, and fluid inertia.[13] The velocity of the particle can be described in the form of relative velocity (u_p/u_l), particle Froude number (Fr_p), or particle generalized Reynolds number (Re_p)

$$\frac{\vec{v}_p}{\vec{v}_l}, \mathrm{Fr_p}, \mathrm{Re_p} = f\left(\mathrm{Re_l}, Fr_l, a, Ar_{l,p}, \frac{D_e}{D}\right), \tag{17.13}$$

where f represents a function of the various dimensionless numbers are $\mathrm{Re_p} = v_p d_e \rho_l/\mu_{ap}$, $\mathrm{Re_l} = v_l D \rho_l/\mu_{ap}$, $\mathrm{Fr_p} = v_p^2/g d_e$, $\mathrm{Fr_l} = v_l^2 g D$, $a = [\rho_p/\rho_l - 1]$, $\mathrm{Ar_p} = a g d_e^3 \rho_l^2/\mu_{ap}^2$, $Ar_l = a g D^3 \rho_l^2/\mu_{ap}^2$.

In the above-defined dimensionless numbers, because of the non-Newtonian character of starch solutions (power-law liquids), the viscosity term is replaced by an apparent viscosity:

$$\mu_{ap} = \frac{2^{(n-3)} m \left[\frac{3n-1}{4n}\right]^n}{\vec{v}_l^{(1-n)} D^{(n-1)}}. \tag{17.14}$$

The stepwise multiple regression of various dimensionless numbers (Equation 17.13) resulted in the following two equations that gave the best fit for the experimental data of particle Froude and Reynolds numbers in the SSHE (R^2 were 0.97 and 0.99, respectively):

$$\mathrm{Fr_p} = 0.23\mathrm{Fr_l^{0.60}}\left(\frac{D_e}{D}\right)^{-0.48}\mathrm{Re_l^{0.50}}\mathrm{Ar_l^{-0.24}}. \tag{17.15}$$

$$\mathrm{Re_p} = 2.69\mathrm{Fr_l^{0.31}}\left(\frac{D_e}{D}\right)^{-0.27}\mathrm{Re_l^{0.23}}\mathrm{Ar_l^{-0.39}}. \tag{17.16}$$

Both Froude and Reynolds numbers were influenced by the particle-to-tube diameter ratio, carrier fluid velocity, density, and viscosity incorporated in different dimensionless numbers. Similar types of correlations were developed for holding tube (i.e., without heating and cooling sections of the aseptic system):

$$\mathrm{Fr_p} = 1.48\mathrm{Fr_l^{0.70}}\left(\frac{D_e}{D}\right)^{-0.65}\mathrm{Re_l^{0.15}}\mathrm{A}r_l^{-0.08}, \tag{17.17}$$

$$\mathrm{Re_p} = 6.65\mathrm{Fr_l^{0.34}}\left(\frac{D_e}{D}\right)^{-0.29}\mathrm{Re_l^{0.10}}\mathrm{Ar_l^{-0.45}}. \tag{17.18}$$

The particle Reynolds number showed a better fit ($R^2 = 0.99$) over the data compared with the Froude number correlation ($R^2 = 0.84$).

Here, $\mathrm{Re_p} = v_p d_e \rho_l/\mu_{ap}$, $\mathrm{Re_l} = v_l D \rho_l/\mu_{ap}$, $\mathrm{Fr_p} = u_p^2/g d_e$, $\mathrm{Fr_p} = v_l^2/gD$, $a = [\rho_p/\rho_l - 1]$, $\mathrm{Ar_p} = a g d_e^3 \rho_l^2/\mu_{ap}^2$, $\mathrm{Ar_l} = a g D^3 \rho_l^2/\rho_{ap}^2$.

Table 17.11 shows the correlations obtained from the generalization of experimental data of some processes related to food and bioprocessing.

17.7 SCALE-UP

Scaling up from laboratory scale to pilot plant or industrial scale is done by achieving similarity with the laboratory scale. There are three types of similarities: geometric similarity, where model and prototype have the same dimension scale ratio; kinematic similarity, where model and prototype have the same velocity scale ratio; dynamic similarity, where model and prototype have the

Table 17.11 Some Correlations Obtained from Dimensional Analysis

Processing Operation	Process Variables	Correlation and Reference
Canning (liquid only) end-over-end rotation	Rotation speed (N), diameter of can (D_c), density of liquid (ρ_l), height of can (H), viscosity of liquid (μ), volume of can (V_c), volume of can headspace (V_p), can angular velocity (ω), surface tension of liquid (σ), thermal conductivity of liquid (k_l)	Duquenoy:[17] $Nu = 17 \times 10^5 Re^{1.45} Pr^{1.19} We^{-0.551} (D_c/2H)^{0.932} (V_p/V_c)^{0.628}$ $Nu = UD_c/2k_l$, $Re = 2\pi N D_c \rho_l L/(D_c + H)\mu$
Canning (liquid only) end-over-end rotation	Rotation speed (N), diameter of rotation (D_r), density of liquid (ρ_l), viscosity of liquid (μ), thermal conductivity of liquid (k_l)	Anantheswaran and Rao:[18] $Nu = 2.9 Re^{0.436} Pr^{0.287}$ $Nu = U(D_r + H)/k_l$, $Re = (D_r + H)^2 N\rho_{1/\mu}$
Canning (heat transfer to liquid in the presence of particles) axial rotation	Rotation speed (N), radius of reel (S), density of liquid (ρ_l), viscosity of liquid (μ), thermal conductivity of liquid (k_l)	Lenz and Lund:[19] $Nu = 115 + 15\, Re^{0.3} Pr^{0.08}$ (single particle in the can) $Nu = -33 + 53\, Re^{0.28} Pr^{0.14} [d_s/S(1-\varepsilon)]^{0.46}$ $Nu = US/k_l$ $Re = S^2 N\rho_l/\mu$
Canning (heat transfer to liquid in the presence of particles) axial rotation	Rotation speed (N), diameter of can (D_c), density of liquid (ρ_l), height of can (H), viscosity of liquid (μ), can angular velocity (ω), particle concentration (ε), drag coefficient (C_D), thermal conductivity of liquid (k_l)	Deniston et al.:[20] $Nu = 1.87 \times 10^{-4} Re^{1.69} [((\rho_p - \rho_l)/C_D\rho_l) ((\omega^2 D_c + 2g)/\omega^2 D_c)(d_s/D_c)]^{0.530} (\alpha_p/\omega D_p^2)^{0.126} [(1-\varepsilon)(H_{ce}/D_{ci}) (\omega D_c^2/\alpha_l)]^{-0.17}$ $Nu = UD_c/k_1$ $Re = \rho_l \omega D_c^2/2\mu$
Canning (heat transfer to particles) axial rotation	Rotation speed (N), density of liquid (ρ_l), viscosity of liquid (μ), thermal conductivity of liquid (k_l), equivalent diameter (d_e), particle sphericity (ψ)	Fernandez et al.:[21] $Nu = 2.7 \times 10^4 Re^{0.294} Pr^{0.33} \psi^{6.98}$ $Nu = h_{fp} d_e/k_l$ $Re = d_e 2_l^{N\rho}/\mu$
Aseptic processing (heat transfer to particles)	Relative velocity between liquid and particle (v_r), density of liquid (ρ_l), viscosity of liquid (μ, K, and n), thermal conductivity of liquid (k_l), particle diameter (d_p), specific heat of liquid (C_{pl}), volumetric thermal expansion coefficient of the liquid (β), average temperature of particle (T_{av})	Baptista et al.:[22] $Nu = Nu_s + 0.17 GRe^{0.71} GPr^{0.42} (d_p/dt)^{0.28}$ $Nu_s = 2 + 0.025 Pr_s^{0.33} Gr^{0.5}$ $Nu = h_{fp} d_p/k_l$ $GRe = 8\rho_l v_r^{2-n} d_p^n/2^n K ((3n+1)/n)^n$ $GPr = C_{pl} K((3n+1)/n)^n 2^{n-3}/k_l (V_r/d_p)^{1-n}$ $Gr = d_p^3 g\beta\rho_l^2 (T_{av} - T_l)/2\mu_s$
Aseptic processing (heat transfer to particles): straight tube	Relative velocity between liquid and particle (v_r), density of liquid (ρ_l), viscosity of liquid (μ, K and n), thermal conductivity of liquid (k_l), particle diameter (d), specific heat of liquid (C_{pl}), volumetric thermal expansion coefficient of the liquid (β)	Ramaswamy and Zeareifard:[23] $Nu = 2 + 3.8 GRe^{0.479} GPr^{-0.655} (d/D)^{2.293} (V_p/V_s)^{0.514}$ $Nu = h_{fp} a/k_l$ $GRe = 8\rho_l v_r^{2-n} d_p^n/2^n K((3n+1)/n)^n$ $GPr = C_{pl} K((3n+1)/n)^n 2^{n-3}/k_l (V_r/d_p)^{1-n}$ $Gr = g\beta\rho_l^2 \Delta T d^3/\mu$
Aseptic processing (fastest particle velocity, v_p): Straight tube	Liquid velocity (v_l), density of liquid and particle (ρ_l and ρ_p), viscosity of liquid (μ, K, and n), particle and tube diameters (d_p, d_t), upward inclination of tube (I)	Baptista et al.:[24] Particle linear velocity (v_p): $V_p/v_l = 0.77 GRe^{0.053} Fr^{0.092} Ar^{0.011} \alpha^{-0.28} (d_p/d_t)^{0.52}$ Particle angular velocity (ω): $\omega/v^l = 0.23 GRe^{0.33} Fr^{-0.25} Ar^{-0.083} \alpha^{0.50} (d_p/d_t)^{0.83}$ $GRe = 8\rho_l v_r^{2-n} d_p^n/2^n K((3n+1)/n)^n$ $Fr = v_l^2/gd_t$ $Ar = (\rho_l\rho_p - \rho_l)\sin(I) g d_p^3/\mu^2$
Mixing (propeller stirrer)	Stirrer speed ($\vec{v}_\theta$), mixing time (θ), height (H), diameter (D)	Zlokarnik:[25] $\vec{v}_\theta \theta \propto (H/D)^{0.85}$ $Re = 10^3$

continued

Table 17.11 continued

Processing Operation	Process Variables	Correlation and Reference
		$\vec{v}_{\theta}\theta \propto (H/D)^{1.5}$ $Re = 10^4 - 10^5$
Heat transfer in bubbling beds	—	Molerus and Wirth:[26] $Nu = 0.165(\mathrm{Ar/Pr})^{(1/3)}$ $10^5 \leq Ar \leq 10^8$ $Nu = 0.02469Ar^{0.4304}$ $\mathrm{Ar} \geq 10^8$
Liquid atomization	—	Dahl and Muschelknautz:[27] $We = 4.5 \times 10^4 Oh^{1/6}$, here Oh is Ohnesorge number ($\mathrm{We}^{1/2}/Re$) Zlokarnik:[5] $We = 1.97 \times 10^4(\vec{v}\mu/\sigma)^{0.154}$

same force scale ratio. Sometimes, partial similarity is achieved when full similarity is not possible in scale-up analysis.[3] Similarity in scale up analysis can be achieved by ensuring that the correlations obtained from small scale equipment after dimensional analysis still hold for larger scale equipment. It is important to note that experiments must cover the full range of values of the relevant dimensionless groups encountered in the scaled-up version.

Scale up of many types of equipments, viz. spray dryer,[28] fluidized bed dryer,[29] spouted bed dryer,[30,31] rotary dryer,[32] pneumatic conveying dryer,[33] layer dryer,[34] mixer granulator,[35] and chemical reactor,[36] as well as industrial processes, viz. spray coating,[37] freeze drying,[38] fermentation,[39] agglomeration,[40] and fluidized beds[41] have been carried out and reported.

17.8 CONCLUDING REMARKS

Application of dimensional analysis is widespread. It is especially useful when a mathematical model is either not possible or not feasible. Certainly, food processing is one of the areas that can make use of dimensional analysis to perform experimental data analysis, model design, prototype testing, and equipment scaling up. In performing dimensional analysis, a relevance list consists of all influencing parameters in a physical problem is made, followed by the generation of π-space that consists of dimensionless numbers. Thereafter, experiments are to be carefully designed and performed. Generalization of experimental data gives relationships and correlations that relate the dimensionless numbers. Based on the relationships and correlations derived empirically, reliable scale up can be carried out.

NOTATION

Symbol	Quantity	Dimension	Unit
Basic physical quantities			
L	Length	[L]	m
t	Time	[t]	s
T	Temperature	[T]	K
M	Mass	[M]	kg
N	Quantity of matter	[N]	mol
Φ	Angle	[Φ]	rad, deg

continued

continued

Symbol	Quantity	Dimension	Unit
Physical quantities			
A	Surface area	$[L^2]$	m^2
a	Density simplex	$[-]$	—
b_t	Vapor expansion in capillaries	$[ML^{-3}T^{-1}]$	$kg\ m^{-3}\ K^{-1}$
c_A	Molar concentration of component A	$[NL^{-3}]$	$mol\ m^{-3}$
c	Heat capacity per unit mass (incompressible substances)	$[L^2t^{-2}T^{-1}]$	$J\ kg^{-1}\ K^{-1}$
c_p	Heat capacity per unit mass at constant pressure	$[L^2t^{-2}T^{-1}]$	$J\ kg^{-1}\ K^{-1}$
c_v	Specific vapor capacity	$[Lt^2M^{-1}]$	$kg\ kg^{-1}\ Pa^{-1}$
D	Diameter	$[L]$	m
D_{AB}	Binary diffusion coefficient	$[L^2t^{-1}]$	$m^2\ s^{-1}$
D	Diameter	$[L]$	m
F	Volumetric flow rate	$[L^3t^{-1}]$	$m^3\ s^{-1}$
g	Gravity acceleration $=9.80665\ m\ s^{-2}$	$[Lt^{-2}]$	$m\ s^{-2}$
G	Mass flow rate per unit area	$[ML^{-2}t^{-1}]$	$Kg\ m^{-2}\ s^{-1}$
H	Height	$[L]$	m
H_e	Magnetic field strength	$[QL^{-2}t^{-1}]$	$C\ m^{-2}\ s^{-1}$
h	Heat-transfer coefficient	$[MT^{-1}t^{-3}]$	$W\ m^{-2}\ K^{-1}$
I	Electric charge flux	$[QL^{-2}t^{-1}]$	$C\ s^{-1}m^{-2}$
k	Thermal conductivity	$[MLT^{-1}t^{-3}]$	$W\ m^{-1}\ K^{-1}$
k_L	Mass-transfer coefficient	$[Lt^{-1}]$	$m\ s^{-1}$
L	Length	$[L]$	m
MR	Moisture ratio	—	—
$\dot{m}$	Mass flow rate	$[Mt^{-1}]$	$kg\ s^{-1}$
n	Specific mass constant	$[MM^{-1}]$	$kg\ kg^{-1}$
P	Pressure	$[ML^{-1}t^{-2}]$	Pa
P_v	Vapor pressure	$[ML^{-1}t^{-2}]$	Pa
Q	Heat released in chemical reaction per mole reacting	$[ML^2t^{-2}N^{-1}]$	$J\ mol^{-1}$
q	Heat flux	$[Mt^{-3}]$	$W\ m^{-2}$
R_A	Molar rate of generation of component A	$[Nt^{-1}L^{-3}]$	$mol\ s^{-1}\ m^{-3}$
R	Radius	$[L]$	m
r	Radius	$[L]$	m
S	Particle area/particle volume	$[L^{-1}]$	$m^2\ m^{-3}$
sw	Specific weight	$[Mt^{-2}L^{-2}]$	$N\ m^{-3}$
T	Temperature	$[T]$	°C, K
t	Time	$[t]$	s
U	Overall heat transfer coefficient	$[MT^{-1}t^{-3}]$	$Wm^{-2}K^{-1}$
V	Volume	$[L^3]$	m^3
$\vec{v}$	Velocity	$[Lt^{-1}]$	$m\ s^{-1}$
$\vec{v}_\theta$	Rotational speed	$[t^{-1}]$	s^{-1}
ν	Kinetic viscosity	$[L^2t^{-1}]$	$m^2\ s^{-1}$
X	Moisture content	$[M\ M^{-1}]$	$kg\ kg^{-1}$
x, y, z	Cartesian coordinate	$[L]$	m
Physical Quantities (Greek Symbols)			
α	Thermal diffusivity	$[L^2t^{-1}]$	$m^2\ s^{-1}$
β	Volumetric thermal expansion coefficient	$[T^{-1}]$	K^{-1}
δ	Thickness	$[L]$	m
ε	Void fraction	—	—
η	Stefan–Boltzmann constant $=5.67\times10^{-8}\ Wm^{-2}K^{-4}$	$[Mt^{-3}T^{-4}]$	$W\ m^{-2}\ K^{-4}$
θ	Mixing time	$[t]$	s
λ	Latent heat of vaporization	$[L^2t^{-2}]$	$J\ kg^{-1}$

continued

continued

Symbol	Quantity	Dimension	Unit
μ	Viscosity	$[ML^{-1}t^{-1}]$	$N\,s\,m^{-2}$
ρ	Density	$[ML^{-3}]$	$kg\,m^{-3}$
σ	Surface tension	$[Mt^{-2}]$	$kg\,m^{-2}$
σ_e	Electrical conductivity	$[Q^2tL^{-3}M^{-1}]$	$C^2\,s\,m^{-3}\,kg^{-1}$
τ	Shear stress	$[ML^{-1}t^{-2}]$	$N\,m^{-2}$
ϕ	Viscous dissipation function	$[t^{-2}]$	s^{-2}
φ	Angle	$[\Phi]$	rad, deg (°)
χ	Mixing quality	—	—
ψ	Sphericity	—	—
Subscripts			
A	Component A		
c	Can		
cd	Characteristic dimension		
cy1	Cylinder		
e	Equivalent		
eff	Effective		
f	Fluid		
fg	Fluid-gas		
fp	Fluid-particle		
g	Gas		
L	Solvent		
l	Liquid		
m	Mass transfer		
0	Initial		
p	Particle		
r	Rotation		
s	Solids		
sat	Saturated		
sph	Sphere		
surr	Surrounding medium		
V	Vapor		
x	Direction in Cartesian coordinate		
y	Direction in Cartesian coordinate		
z	Direction in Cartesian coordinate		
∞	Infinity		

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