ME 6204

Convective Heat Transfer

A STUDY OF THE EFFECT OF DROPLET TEMPERATURE DURING ICE SLURRY FORMATION

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Introduction and Statement of the Problem

Consider an experimental test section filled with water and a *coolant immiscible in water*. Due to its *higher density*, the coolant is at the bottom part of the test section. This coolant is circulated by a pump through the cold bath which *lowers the coolant temperature far below* $0^{\circ}C$ and then it is injected from the top of the section. At this stage, *ice is formed around the injected liquid droplet surface*.

A one-dimensional model for ice layer growth around the injected super-cooled coolant droplets is to be developed,

(a) to predict the growth rate of the ice layer on the droplet surface, and

(b)to find the corresponding temperature profile within the layer.

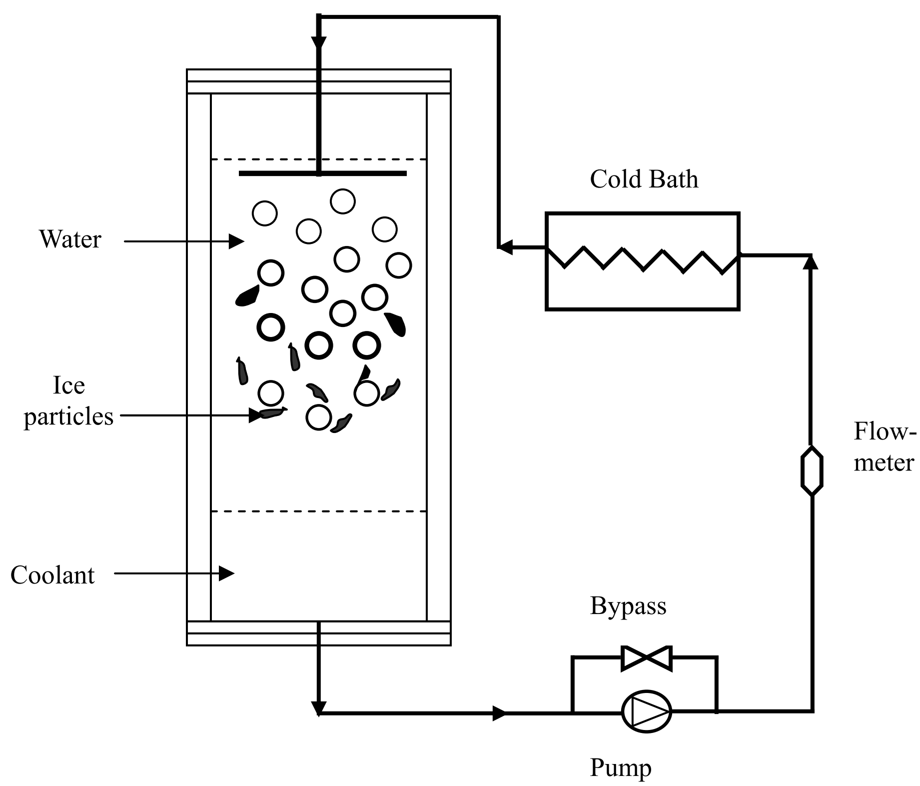

Fig. Schematic diagram of the experiment being modeled

CHAPTER 1

1.1 INTRODUCTION

A system has been developed for the generation of ice slurry using direct contact heat transfer between water and the coolant, Fluroinert FC-84. A one-dimensional model for ice slurry growth around the injected super-cooled coolant droplets in the ice generator is developed. The growth of ice and mushy region around the supercooled liquid droplets involves phase change and heat conduction between these layers. These conditions reduce the ice thickness problem to a differential equation, from which other quantities, such as, the mushy region thickness and temperature profiles in the layers can be determined. The simultaneous solutions of these equations for different injecting droplets temperatures predict the growth rate of ice and mushy regions, and the corresponding temperature profiles of these layers. In this paper we present the model and its numerical solution.

1.2 LITERATURE REVIEW

Recently, attention has been paid actively to develop an ice thermal energy storage system for district cooling for the purpose of energy saving and reducing electrical costs, specially during peak hours in the summer [1,2]. In particular, it would be beneficial to utilize the large latent heat of ice for more efficient cooling purposes. In a ice storage systems, due to the insulating characteristics of ice, the freezing heat-transfer performance decreases due to buildup of ice layers on heat exchanger surfaces during storage of cool energy [3]. To resolve this problem, a novel thermal energy storage system with ice water slurry has been developed [4,5]. Ice slurry, which consists of fine ice crystals and liquid water, has a large latent heat of fusion. Moreover, cooling capacity of slurry ice is about four to six times higher than that of conventional chilled water system [6].

Several techniques for ice slurry formation by direct contact heat transfer have been developed recently. Chen et al. [7] investigated the nucleation probability of super cooled water inside cylindrical capsules. Song et al. [8] presented a simple model on direct contact heat transfer between two immiscible liquids in a countercurrent spray column and obtained the numerical solution with the variable step Runge-Kutta method.

Wijeysundera et al. [9] studies the generation of ice slurry using direct contact heat transfer between water and injecting super cooled liquid Fluroinert (FC-84) and estimates the heat transfer co-efficient between liquid droplets and water. These literatures reveal that several methods of producing ice slurries exist, though many of these need to be investigated in greater detail.

1.3 OBJECTIVES

The main motivation of present study is to understand the physical phenomena of ice and mushy layers growth around the super cooled liquid droplet. The results obtained from this study and techniques employed will help in the design and analysis of a system for the production and storage of ice slurry using direct contact heat transfer between water and coolant.

CHAPTER 2

2.1 ASSUMPTIONS

The assumptions made in the following analyses are as follows:

(i) the effects of turbulence, radiation, splashback of droplets are negligible;

(ii) convection coefficient and thermophysical properties are constant;

(iii) conduction of heat through the both layers, ice and mushy, is considered.

2.2 MATH MODEL

Stefan's [10] one-dimensional mathematical approach is used to model the ice and unfrozen water layer growth. In the following analysis, one-dimensional model will be considered. The configuration is depicted in Fig. 1, where B(t) and b(t) are ice layer and mushy layer thickness, respectively.

Heat transfer during ice-slurry formation with coolant involves three-phase conditions, including multiphase flow with droplets, phase change and heat conduction in the ice and mushy layer.

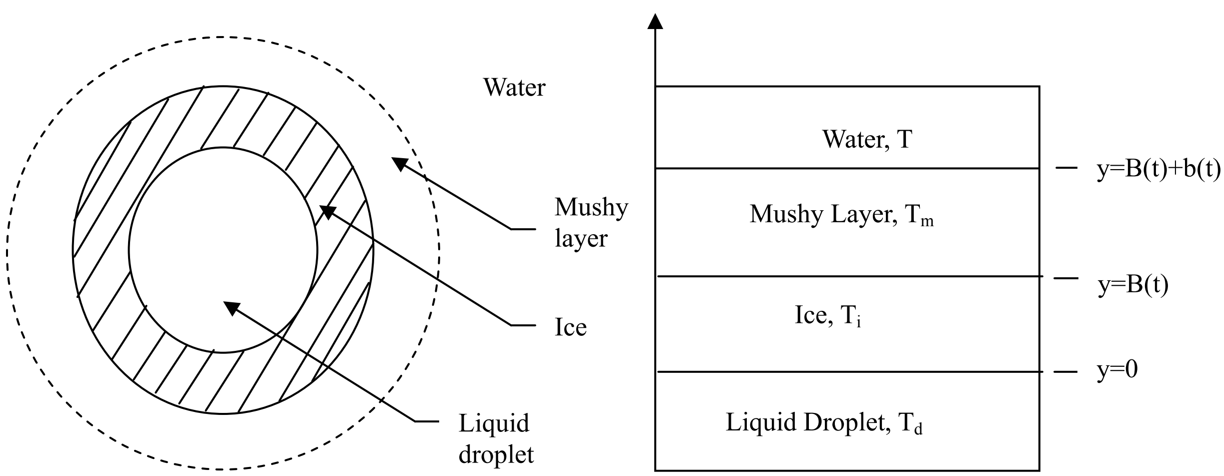

Fig 1 (a). Schematic of of the liquid droplet-ice-mushy-water layers (b). Cross-section of layers

The growth process of ice layer and mushy layer around the impinging super-cooled liquid droplets occurs in two distinct stages- at first stage, water comes in contact with the super cooled liquid droplets and freezes instantaneously, while at the second stage, both ice and mushy layer develop simultaneously. The initial incoming liquid droplet must, therefore, immediately adopt the sub-zero temperature and since there are nucleation site, freeze around the droplet.

Let the temperature in the liquid droplet be denoted by $T_d(t)$, while the temperatures in the ice and mushy layers are denoted by $T_i(y, t)$ and mushy layer is $T_m(y, t)$, respectively.

2.2.1 Governing Equations:

The governing equations that describe heat transfer and ice formation are described in this section.

In the liquid droplet region, it is assumed that due to the formation of ice layer around the liquid droplet, the latent heat is conducted through the ice layer to the liquid droplet.

Then, the energy balance is given by,

Energy gained by the liquid = Heat transfer by conduction to droplet during dt

$$(mC)_d \, dT_d = -k_i A \frac{dT}{dy} dt \qquad (1)$$

where, m, C, A are the mass, specific heat and surface area, subscript d and i denote droplet and ice layer.

In the ice layer region, the heat diffusion equation is,

$$\frac{\partial T_i}{\partial t} = \frac{k_i}{\rho_i C_i} \frac{\partial^2 T_i}{\partial y^2} \qquad (2)$$

In the mushy layer region, the heat diffusion equation is,

$$\frac{\partial T_m}{\partial t} = \frac{k_m}{\rho_m C_m} \frac{\partial^2 T_m}{\partial y^2} \qquad (3)$$

where, k, ρ, C are the thermal conductivity, density and specific heat, subscript i and m denote ice and mushy layer.

Again, from the Stefan condition, the velocity of the moving ice/mushy layers interfaces, B(t), is proportional to the heat flux across it.

$$\rho_i L \frac{\partial B}{\partial t} = k_i \frac{\partial T_i}{\partial y} - k_m \frac{\partial T_m}{\partial y} \qquad (4)$$

which states that the velocity of the interface is proportional to the heat flux across it; L is the latent heat of fusion.

Furthermore, the growth of ice and mushy layers depend on the incoming liquid droplets velocity, cooling and geometry factor [11].

$$\rho_i \frac{\partial B}{\partial t} + \rho_m \frac{\partial b}{\partial t} = \alpha \gamma V G \qquad (5)$$

2.2.2 Boundary Conditions:

During the initial stage (t = 0), when there is only ice growth around the liquid droplet, the initial ice layer thickness can be found by integrating equation (5), with mushy layer thickness set to zero, to give,

$$B = \left(\frac{\alpha \gamma V G}{\rho_i}\right) t \qquad (6)$$

The liquid droplet temperature profile can be calculated by integrating equation (1) subject to T_{di} be initial droplet temperature and T_d be the final temperature.

$$T_d = T_i + (T_{di} - T_i) e^{-\frac{\lambda t}{B}} \qquad (7)$$

where, $\lambda = \dfrac{6k_i}{\rho_d C_d D}$

The time taken for an incremental ice thickness growth is smaller than the time taken for the heat to be fully conducted through the newly formed layer. It is then approximated that the layer growth is sufficiently slow that heat looses/gains at the phase interface are fully conducted throughout the solid.

Then, heat conduction within the ice layer as well as the mushy layer, can be approximated as quasi-steady process [12]. The governing heat equation (2) for the ice then becomes,

$$\frac{\partial^2 T_i}{\partial y^2} = 0 \qquad (8)$$

subject to boundary conditions of $T_i(0,t) = T_d$ at $y = 0$, $t > 0$ and at interfacial condition of $T_i(B,t) = T_f$ at $y = B$, $t > 0$. Solving Eq. (8) subject to the boundary conditions,

$$T_i(y,t) = \left(\frac{T_f - T_d}{B}\right) y + T_d \qquad (9)$$

This equation represents a quasi-steady result since the ice thickness, B, varies with time.

In an analogous manner for the mushy layer, equation (3) reduces to

$$\frac{\partial^2 T_m}{\partial y^2} = 0 \qquad (10)$$

subject to boundary conditions of $T_m(B,t) = T_f$ at $y = B, \quad t > 0$. An energy balance at the ice/mushy layer interface, $y = B + b, \quad t > 0$, yields the following interfacial constraint,

$$k_m \frac{\partial T_m}{\partial y}\Big|_{y=B+b} = h(T_m\Big|_{y=B+b} - T) \qquad (11)$$

where, the term represents conduction through the mushy layer (left hand side) is balanced by the the heat flux by convection (right hand side)through this region.

The expression for heat conduction in Eq.(11) depends on the slope of the temperature profile in the mushy layer. However, since the mushy layer temperature is expected to be very close to T_f, a single slope of temperature profile for the layer can be assumed in the present analysis. There exists a lack of relevant experimental data due to the difficulty of reliable temperature measurements over such a small distance within the mushy layer. For this reason, it is considered that the equilibrium freezing temperature in the mushy layer. Then, consider the temperature is at the freezing point,

$$T_i(B,t) = T_m(0,t) = T_f \qquad (12)$$

Solving Eq. (11), subject to the boundary conditions,

$$T_m = T_f + \frac{(T - T_f)}{(b - \frac{k_m}{h})}(y - B) \qquad (13)$$

This represents a quasi-steady temperature profile of the mushy layer, whereby the mushy layer temperature profile varies with time since the growth of both ice layer, $B(t)$ and mushy layer, $b(t)$, are time dependent.

Now, the unknowns, $B(t)$ and $b(t)$, can be determined by the simultaneous solutions of Eq.(4) and Eq.(5).

To calculate $B(t)$, solving the energy balance equation, Eq.(4),at the ice/mushy layer interface, $y = B$, gives to

$$\frac{dB}{dt} = \frac{u_0}{B} - \frac{u_1}{b - \frac{k_m}{h}} \tag{14}$$

where,

$$u_0 = \frac{k_i (T_f - T_d)}{\rho_i L}$$

$$u_1 = \frac{k_m (T - T_f)}{\rho_i L}$$

First term on the right hand side represents the heat conduction through the ice. This indicates that the ice growth rate decreases with the increase of the temperature of the liquid droplet and the increase of the ice layer thickness. The second term on the right hand side is proportional to u_1 which incorporates the mushy layer. It indicates that the mushy layer thickness plays to the ice growth rate; with the increase of the mushy layer, ice growth rate decreases. Again, as h decreases, the denominator of the second term becomes negligible and the ice growth rate decreases significantly. On the other hand, increase of h significantly has a little effect to the ice growth rate. This indicates the relation to the ice growth and the mushy layer.

Calculating $b(t)$, solving the equation, Eq.(4), gives to

$$\frac{db}{dt} = \frac{\alpha \gamma V G}{\rho_m} - \frac{\rho_i}{\rho_m} \frac{dB}{dt} \tag{15}$$

This equation shows that the mushy layer growth rate depends on the velocity of the liquid droplet and the growth rate of the ice layer. When ice growth rate increases, mushy layer growth rate decreases.

The results in Eq.(15) can now be substituted into Eq.(14) and subsequently solved for $B(t)$. In this way, Eqs.(14) and (15) are solved for $B(t)$ and $b(t)$ sequentially. Since the equations are non-linear, coupled differential equations, the solution will be obtained numerically.

2.3 NUMERICAL SOLUTION

Runge-Kutta scheme, an explicit method, is used for the numerical discretization of the ordinary differential equations. In this analysis, 4-stage Runge-Kutta method is adopted.

From stability analysis, it is found that the stability of this method is less than 2.88, the CFL condition for the Runge-Kutta method. Due to the explicit method, this scheme has higher accuracy than other algebraic type numerical solutions (implicit methods). Moreover, for higher stage, 4-stage Runge-Kutta method has better accuracy than other explicit methods such as lower stage Runge-Kutta methods or the Euler method.

CHAPTER 3

3. RESULTS AND DISCUSSION

The formation of ice and mushy layers are observed on super cooled liquid droplet injected under different temperatures. The development of ice and mushy layers around the liquid droplet depend on the initial injecting temperature, T_{di}.

Case1: $T_{di} = -5^\circ C, D = 4mm, V = 0.21m/s$

Case2: $T_{di} = -10^\circ C, D = 4mm, V = 0.21m/s$

Case3: $T_{di} = -15^\circ C, D = 4mm, V = 0.21m/s$

Fig. 2 shows the growth of ice layer over a period of time when the initial liquid droplet temperatures, T_{di} are $-5^\circ C$, $-10^\circ C, -15^\circ C$, respectively.

For case 1, curve shows a rapid growth of ice layer for the first 0.2 seconds, after which the growth rate becomes almost steady. It continues to grow until the ice thickness is 0.628 mm at 3.5 sec. It is also observed that like case 1, ice growth rate for the first 0.2 seconds after injection of coolant is very fast, though maximum ice thickness is 1.792mm for case3, while for case2, it is 1.396mm.

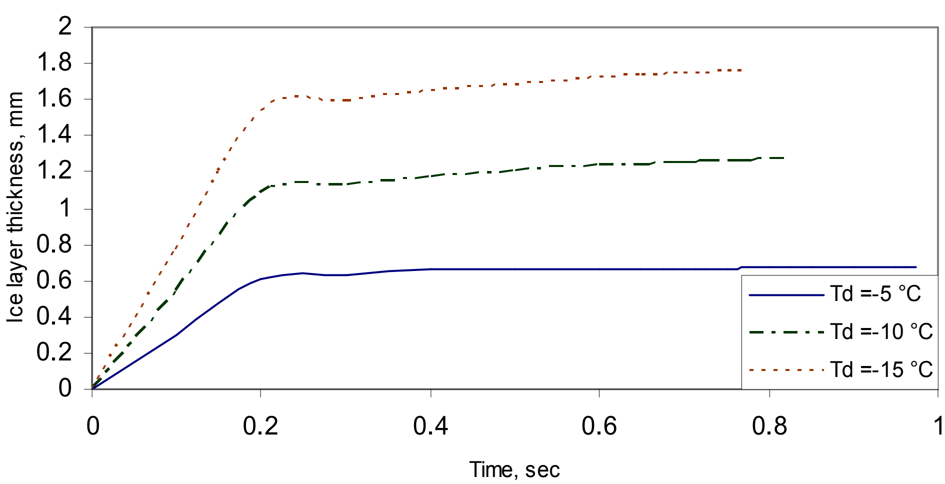

Fig. 2. Ice layer growth for different T_d

Ice grows initially only around the injected super cooled liquid droplet, after which both ice and mushy layer develops simultaneously. That is why ice growth rate becomes almost flat after few seconds of the operation.

Again, the differences among the ice thickness and growth rate time are due to the choice of different initial temperatures. As all other parameters are kept constant, ice growth rate

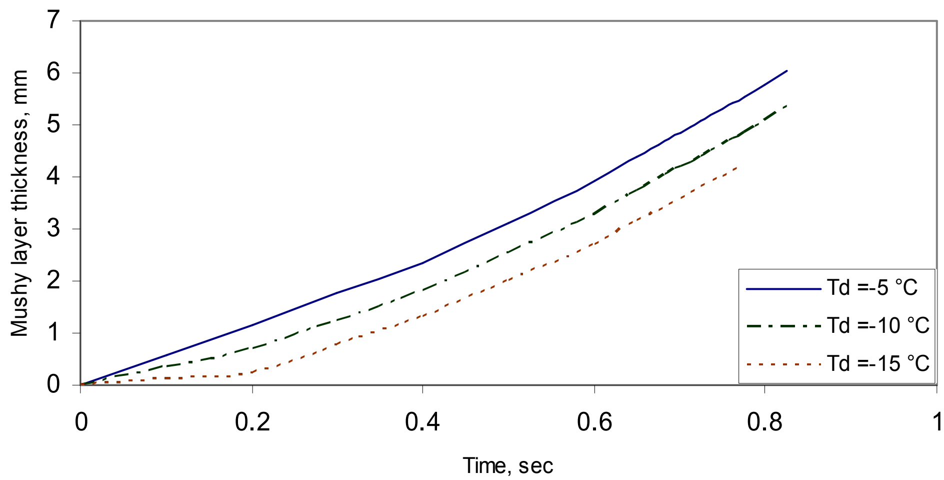

Fig. 3. Mushy layer growth for different T_d

is inversely proportional to the increase of liquid droplet temperature. However, this growth rate is not constant over temperatures. For decreasing temperature from $-5^{\circ}C$ to $-10^{\circ}C$, ice thickness increases by about 55%, whereas for temperature decreasing from $-10^{\circ}C$ to $-15^{\circ}C$, growth of ice thickness is about 22%.

Growth rate of mushy layer, consists of very fine ice particles and water, around the ice layer is shown in Fig.3. For all three cases, it is observed that growth of mushy layer increases with time.

After initial ice growth, mushy layer begins to develop around the ice layer. As a result, it is observed that ice layer growth rate becomes almost steady while the mushy layer grows linearly.

It is also observed from Fig. 3 that the thickness of the mushy layer is maximum (5.9mm) for the liquid droplet injecting at $-5^{\circ}C$. It reveals that mushy layer thickness increases with the increase of liquid droplet temperatures. In other words, mushy layer growth rate is inversion to the ice layer growth rate.

Fig. 4. shows that the liquid droplets temperatures increases exponentially, as stated in

Eq.(1), which is due to the growth of ice layer and mushy layer simultaneously around it. For all three cases, there is a sharp decrease of liquid temperature after being steady for a while. This indicates that the termination of the surrounding ice and mushy layers growth around the liquid droplet at that time.

From the Fig. 4, it is also observed that the residence time is maximum (10 sec) for case 3, while it is minimum, 3.5 sec, for case 1. That is why it can be said that the residence

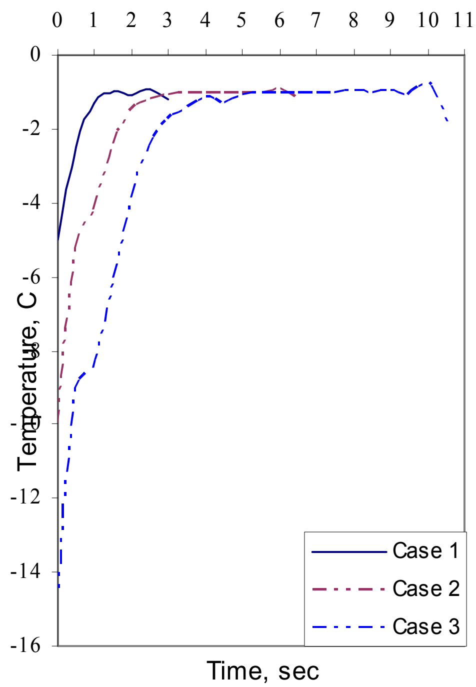

Fig. 4. Liquid droplet temperatures at different conditions

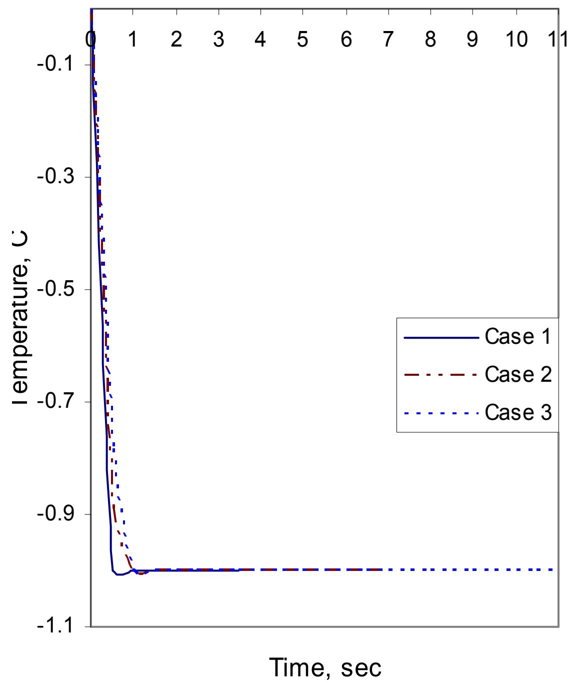

Fig. 5. Mushy layer temperature

times for growth of ice and mushy layers are proportional to the decreasing droplet temperatures.

Also, as observed from Fig. 5, there is a temperature gradient inside the mushy layer which ensures the presence of ice particles in that region.

Although this study has attempted to derive analytical model for ice layer and mushy layer buildup on the surface of liquid droplets, the sensitivity and dependence of this model on various coefficients should not be overlooked. For example, the convection coefficient (typically accurate within $\pm 25\%$). In spite of these limitations, this study gathers the influence of the initial temperature of the liquid droplet and generation of ice around it. As a result, a deeper physical insight of the growth of ice and mushy layers has been provided in this article.

CHAPTER 4

CONCLUSION

In this article, a simplified mathematical model has been developed to predict the simultaneous growth of ice and mushy layers around the surface of the liquid droplet. The analysis includes the solutions of lumped system inside the droplet, and conduction equations for thin ice and mushy layers. The results show that ice generation is faster just after dispersing from the nozzle while mushy layer growth around the ice layer is steady. Furthermore, both ice and mushy layer thickness depend on the inlet coolant temperature. The ice slurry formation rate depends on the inlet coolant temperature. This model therefore provides significant information to understand the physical phenomena of ice slurry generation which may facilitate the development of improved design and operational guidelines for general implementations of the ice slurry cooling systems.

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APPENDIX A

Table 1:
Parameter values used for Figs.2-5

α	0.9	
C_d	1050	J kg^{-1} K^{-1}
D	0.004	m
G	1730	kg m^{-3}
γ	1.0	
h	250	W m^{-2} K^{-1}
k_i	2.18	W m^{-1}K^{-1}
k_m	0.52	W m^{-1}K^{-1}
ρ_d	1730	kg m^{-3}
ρ_i	917	kg m^{-3}
ρ_m	1000	kg m^{-3}
L	3.344×10^5	J kg^{-1}
T_f	-1	ºC
T	0	ºC
V	0.21	m s^{-1}

APPENDIX B

Nomenclature

B	thickness of ice layer (m)
b	thickness of mushy layer (m)
C	specific heat (J kg^{-1} K^{-1})
D	drop diameter (m)
G	liquid content (kg m^{-3})
h	convective heat transfer co-efficient (W m^{-2} K^{-1})
k	thermal conductivity (W m^{-1}K^{-1})
L	latent heat (J kg^{-1})
T	temperature of layers (ºC)
t	time (sec)
V	velocity of liquid droplet (m s^{-1})

Greek symbols

α	cooling factor
γ	surface curvature factor
λ	time constant (s^{-1})
ρ	density (kg m^{-3})

Subscripts

d	droplet
m	mushy layer
i	ice layer
f	ice/mushy layer interface