Problem 1

Water evaporating from a pond does so as if it were diffusing across an air film 0.15 cm thick. The diffusion coefficient of water in 20º C air is about 0.25cm^2/sec. If the air out of the film is fifty percent saturated, how fast will the water level drop in a day?

Given:

Air temperature, T_a = 20º C
Air pressure, $P_a = 0.5\ P_{sat}$
Diffusion Coefficient, D = 0.25 cm^2/sec
Air film thickness, l = 0.15 cm

To Find:

Evaporation of water vapor, j_{H_2O} = ?

Figure:

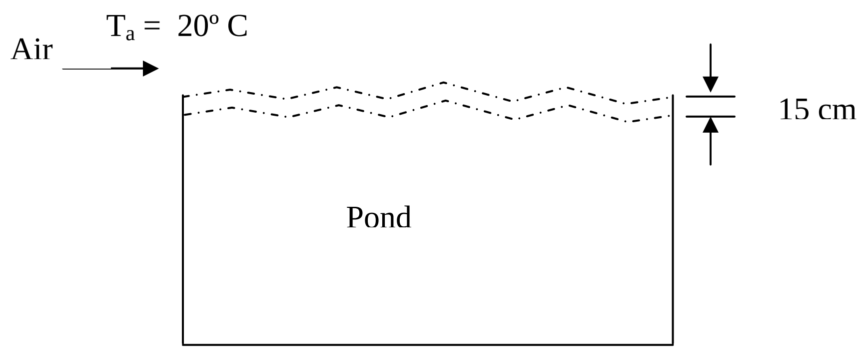

Figure 1: Diffusion on water pond

Solution:

From the air property,

At T_a = 20º C, P_{sat} = 2.34 Pa [From Saturated Water Property]

Now,
Mass flux of water vapor,

$$j_{H_2O} = \frac{D}{l}\Delta c \qquad \text{(i)}$$

But,

$$\Delta c = \frac{P_{sat} - P_a}{RT} \qquad \text{(ii)}$$

where,

Universal Gas Constant, R = 8.314 kJ/kmol-K

Substitute the values in equation (ii), we get,

$$\Delta c = \frac{P_{sat} - P_a}{RT}$$

$$= \frac{P_{sat} - 0.5P_{sat}}{RT}$$

$$= \frac{0.5 \times 2.34}{8.314 \times (273 + 20)} \quad \frac{kPa}{\frac{kJ}{kmol - K} \times K}$$

$$= 4.8 \times 10^{-3} \times 10^{3} \quad \frac{Pa}{\frac{J}{mol}}$$

$$= 0.48 \qquad \frac{N/m^2}{\frac{N - m}{mol}}$$

$$= 0.48 \qquad mol/m^3$$

$$= 0.48 \times 10^{-6} \quad mol/cm^3$$

$$= 18 \times 0.48 \times 10^{-6} \quad cm^3/cm^3 \qquad [\because 1\text{mol} = 18\ cm^3, \text{ for water}]$$

$$= 18 \times 0.48 \times 10^{-6}$$

Substitute the values in equation (i), we get,

$$j_{H_2O} = \frac{D}{l}\Delta c$$

$$= \frac{0.25}{0.15} \times 18 \times 0.48 \times 10^{-6} \quad \frac{cm^2/s}{cm}$$

$$= \frac{0.25}{0.15} \times 18 \times 0.48 \times 10^{-6} \times 3600 \times 24 \quad \frac{cm}{s} \times \frac{s}{hr} \times \frac{hr}{day}$$

$$= 1.24\ cm/day$$

Problem 2

In 1765, Benjamin Franklin made a variety of experiments on the spreading of oils on the pond in Clapham Common, London. Franklin estimated the thickness of the oil layers to be about 25 angstroms. Many more recent scientists have tried to use similar layer of fatty acids and alcohols to retard evaporation from ponds and reservoirs in arid regions. The monolayer used today usually are characterized by a resistance around 2 seconds per centimeter. Assuming that they are the thickness of Franklin's layer and that they can dissolve up to 1.8% water, estimate the diffusion coefficient across the monolayer.

Given:

Solubility of water in oil, S = 1.8%
= 0.018
Monolayer thickness, l = 25A = 25×10^{-8} cm

Resistance of monolayer, Res = 2 sec/cm

To Find:

Diffusion coefficient across the monolayer, D = ?

Assumption:

(1) Monolayer is used during the experiment.

Solution:

Permeability of film, P = $\dfrac{1}{\mathrm{Re}\,s}$

$$= \frac{1}{2} \quad \frac{1}{\text{sec/cm}}$$

$$= 0.5 \quad \text{cm/sec}$$

Now,

$$P = \frac{DS}{l}$$

Or, $$D = \frac{Pl}{S}$$

Or, $$D = \frac{0.5\times25\times10^{-8}}{0.018} \quad \frac{cm}{\text{sec}}\times cm$$

Or, $$D = 6.94\times10^{-6} \quad cm^2/\text{sec}$$

Problem 3

The diffusion coefficient of NO_2 into stagnant water can be measured with the apparatus shown in the figure. Although the water is initially pure, the mercury drop moves to show that 0.82 cm^3 of NO_2 is absorbed in 3 minutes. The gas-liquid interface has an area of 36.3 cm^2, the pressure is 0.93 atmosphere, the temperature is 16º C, and the Henry's law constant is 37000 cm3-atm/mol. What is the desired diffusion coefficient?

Given:

Water is initially pure. (Assumption)
Volume of NO_2 = 0.83 cm^3
Time of NO_2 absorbed, t = 3 min = 180 sec
Gas-liquid,
interface area, A = 36.3 cm^2
pressure, P = 0.93 atm
temperature, T = 16º C
Henry's law constant, H = 37000 cm^3 –atm / mol

To Find:

Diffusion coefficient, D = ?

Figure:

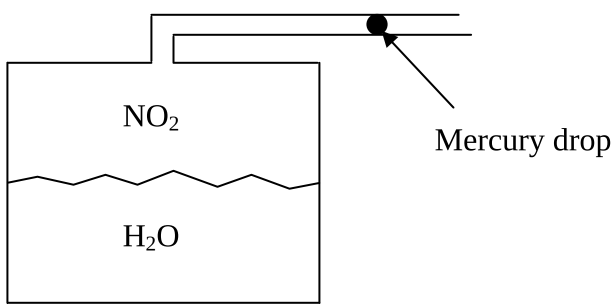

Figure 2: Apparatus Setup

Assumption:

(1) Diffusion of NO_2 into the stagnant water can be considered as the diffusion in a semi-infinite slab.

(2) Diffusion of NO_2 into the stagnant water is unsteady.

Solution:

According to the assumption, the semi-infinite slab contains a uniform concentration of solute $c_{1\infty}$.

The general solution of the diffusion of flux in semi-infinite slab,

$$j_1 = -D\frac{\partial c_1}{\partial z} = \sqrt{D/\pi t}\; e^{-z^2/4Dt}(c_{10} - c_{1\infty}) \qquad \text{(i)}$$

where, c_{10} and $c_{1\infty}$ are the concentrations of solute.

Now, at the interface, z = 0, $c_{10} = c\, x_1 = \frac{p_{10}}{H}$

$c_{1\infty} = 0$

Where,

c = total molar concentration in the liquid

x_1 = mole fraction

p_{10} = partial pressure in the gas phase

Then, from equation (i) becomes,

$$j_1 = \sqrt{\frac{D}{\pi t}}\,(c_{10}) \qquad \text{(ii)}$$

This flux is the value at the particular time t and not that averaged over time.

Now,

$$c_{10} = \frac{p_{10}}{H}$$

$$= \frac{P - P_{air-water}}{H}$$

$$= \frac{0.93 - 0.019}{37000} \quad \frac{atm}{\frac{cm^3 - atm}{mol}}$$

$$= 2.46\times 10^{-5} \text{ mol/cm}^3$$

Now, flux, $j_1 = \dfrac{Volume}{area} \times Concentration$

$$= \frac{Volume}{area} \times \frac{P}{RT}$$

$$= \frac{0.82}{0.36} \times \frac{0.93}{8.205 \times 10^{-2} \times (273+16)} \quad \frac{cm^3}{cm^2} \times \frac{atm}{\dfrac{atm-m^3}{kmol-K} \times K}$$

$$= \frac{0.82}{0.36} \times \frac{0.93}{8.205 \times 10^{-2} \times (273+16)} \quad cm \times \frac{atm}{\dfrac{atm-(100)^3 cm^3}{(1000)mol}}$$

$$= \frac{0.82}{0.36} \times \frac{0.93}{8.205 \times 10^{-2} \times (273+16)} \times \frac{1000}{100^3} \quad \frac{mol}{cm^2}$$

$$= 8.86 \times 10^{-7} \quad \frac{mol}{cm^2}$$

Then, to calculate flux of NO_2 into the stagnant water over average time, equation (ii) becomes,

$$\frac{dj_1}{dt} = \sqrt{\frac{D}{\pi t}}\ c_{10}$$

Integrating,

$$j_1 = 2\sqrt{\frac{D}{\pi}}\ t^{1/2}\ c_{10} + C$$

At, $t = 0$, $j_1 = 0$, $C = 0$

Then,

$$j_1 = 2\sqrt{\frac{D}{\pi}}\ t^{1/2}\ c_{10}$$

Substitute the values in the equation, we get,

$$8.86 \times 10^{-7} \quad \frac{mol}{cm^2} = 2\sqrt{\frac{D}{\pi}}\ (180)^{1/2} (2.46 \times 10^{-5})\ (\text{sec})^{1/2} \ \frac{mol}{cm^3}$$

or, $D = 5.51 \times 10^{-6} \ \dfrac{cm^2}{\text{sec}}$