Home Assignment Problem for ME6203- Mass Transport

By Tang, Yun; Xiong, Jie HT030785E Group number 5

Chapter 3-9: Copper dispersed in porous low-grade ore pellets 0.2cm in diameter is leached with 4-M H_2SO_4.The copper dissolves quickly, but diffuses slowly out of the pellets. Because the ore is low grade, the porosity can be assumed constant, and the copper concentration will be low in the acid outside of the pellets. Estimate how long it will take to remove eighty percent of the copper if the effective diffusion coefficient of the copper is $2.6 \cdot 10^{-6}$ cm²/sec.

Solution: The physics here is similar with that of Example 3.5-3. With the pellet, a mass balance on a spherical shell yields

$$\frac{\partial c_1}{\partial t} = D_{eff} / r^2 \frac{\partial}{\partial r}(r^2 \frac{\partial c_1}{\partial r})$$

This equation is subject to

t=0, for all r, $c_1 = c_{10}$

t>0, r=0, $((\partial c_1)/(\partial r))=0$

r= R_0, $c_1 = C_{sat}$ (t),

$r = \infty, C_1 = 0$

Where R_0 is the pellet radius and C_{sat} is the saturation concentration at the surface of pellet.

Because the solvent is not well stirred and the copper diffuses slowly, the concentration far from the pellet is zero.

Now we make a mass balance on the solute in the bath of volume V_B

$$V_B \frac{dC_1}{dt} = -4\pi R^2 D_{eff} \frac{\partial c_1}{\partial t} |_{r=R_0}$$

Subject to t=0, C_1=0.

The mathematical solution to this problem is given as

$$C_1 = \frac{c_{10}}{1+B} - 6V_B \sum_{n=1} \frac{e^{-D_{eff}\alpha_n^2 t}}{B^2 R_0^2 \alpha_n^2 + 9(B+1)}$$

In which $\tan(R_0\alpha_n) = \dfrac{3R_0\alpha_n}{3 + BR_0^2\alpha_n^2}$

and $B = \dfrac{V_B}{(4/3)\pi R_0^3 \varepsilon}$, where ε is the void fraction in the sphere.

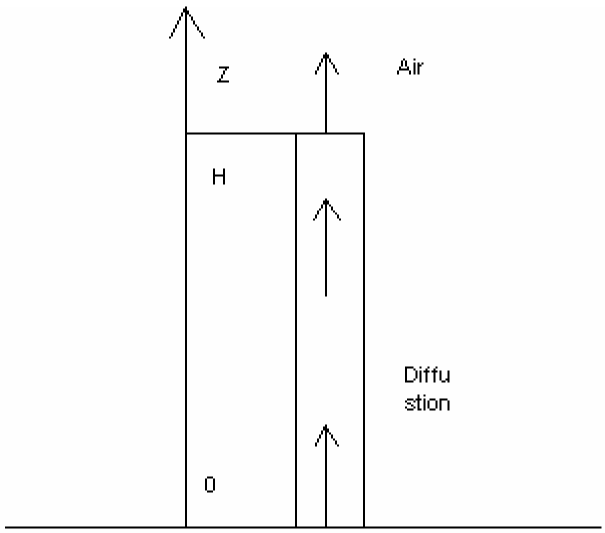
We then calculate consider $B = \dfrac{V_B}{(4/3)\pi R_0^3 \varepsilon}$=1, (Consider one pellet only and as the copper concentration outside the pellet is low)

Therefore we have B=1,

When 80% copper is removed, this condition could not give me the exact number for $C_1\,(1+B)/c_{10}$, as the bath concentration C_1 cannot be calculated.

Chapter 3-10 A large polymer slab initially containing traces of solvent is exposed to excess fresh air to allow the solvent to escape. Find the concentration of solvent in the slab as a function of position and time. Assume that the diffusion coefficient is a constant, but discuss how you might expect it to vary.

Solution:



As the above picture shows, the diffusion is one-dimensional only inside the slab in Z direction across the slab, while we consider anywhere in the slab cross-section is the same situation. So what we want to find is the concentration profile in the z direction.

Governing Equations:

$\dfrac{\partial c_1}{\partial t} = D\dfrac{\partial^2 c_1}{\partial z^2}$, assuming the diffusion coefficient is constant.

This equation is subject to the following conditions:

t=0, all z $c_1 = c_{10}$ (initial concentration in the slab)

t>0, z=H, $c_1 = 0$ this is because the slab is exposed to outside air and the surface solvent diffuses quickly into the air.

Mathematical Solution:
The solution of this problem depends on using the combination of variables. A new variable

$$\zeta = \frac{z - H}{\sqrt{4Dt}}$$

The differential equation can then be written as

$$\frac{dc_1}{d\zeta}\left(\frac{\partial \zeta}{\partial t}\right) = D\frac{d^2 c_1}{d\zeta^2}\left(\frac{\partial \zeta}{\partial z}\right)^2$$

Or $\frac{d^2 c_1}{d\zeta^2} + 2\zeta \frac{dc_1}{d\zeta} = 0$

The boundary conditions are given as

$$\zeta = 0, c_1 = 0$$

$$\zeta = \infty, c_1 = c_{10}$$

The solution is now straightforward. One time integration of the equation gives

$\frac{dc_1}{d\zeta} = ae^{-\zeta^2}$, where a is an integration constant. A second integration and use of boundary conditions gives

$$\frac{c_1 - 0}{c_{10} - 0} = erf\zeta \text{ ,}$$

Where

$erf\zeta = \frac{2}{\sqrt{\pi}}\int_0^{\zeta} e^{-t^2}dt$, which is the error function of ζ

The above result is obtained based on the constant diffusion coefficient assumption. In real situation, the diffusion coefficient is expected to drop as the evaporation will cause the temperature to drop.